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The cost-effectiveness relationships for various agricultural and desalination alternatives for controlling salinity in irrigation return flows are developed. Selection of optimal salinity management strategies on a river basin scale is described as a problem of integrating optimal strategies with individual subbasins and irrigated valleys.

Desalination systems include seven processes: (1) multi-stage distillation; (2) vertical tube evaporation in conjunction with (1); (3) a vapor compression form of (2); (4) electrodialysis; (5) reverse osmosis; (6) vacuum freezing - vapor compression; and (7) ion exchange. Agricultural salinity control alternatives include conveyance linings, irrigation scheduling, automation, sprinkler irrigation systems, and trickle irrigation systems.

A case study of the Grand Valley in western Colorado is presented to demonstrate the analysis developed. Results indicate that treatments of the agricultural system are generally more cost-effective than desalting except for high levels of potential salinity control. Lateral linings and on-farm improvements are the best agricultural alternative.

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INTEGRATING DESALINATION AND AGRICULTURAL SALINITY CONTROL ALTERNATIVES

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FOREWORD

The Environmental Protection Agency was established to coordinate administration of the major Federal programs designed to protect the quality of our environment.

An important part of the Agency's effort involves the search for information about environmental problems, management techniques and new technologies through which optimum use of the Nation's land and water resourcescan be assured and the threat pollution poses to the welfare of the American people can be minimized.

EPA's Office of Research and Development conducts this search through a nationwide network of research facilities.

As one of these facilities, the Robert S. Kerr Environmental Research Laboratory is responsible for the management of programs to: (a) investigate the nature, transport, fate and management of pollutants in groundwater; (b) develop and demonstrate methods for treating wastewaters with soil and other natural systems; (c) develop and demonstrate pollution control technologies for irrigation return flows; (d) develop and demonstrate pollution control technologies for animal production wastes; (e) develop and demonstrate technologies to prevent, control or abate pollution from the petroleum refining and petrochemical industries; and (f) develop and demonstrate technologies to manage pollution resulting from combinations of industrial wastewaters or industrial/municipal wastewaters.

This report contributes to the knowledge essential if the EPA is to meet the requirements of environmental laws that it establish and enforce pollution control standards which are reasonable, cost effective and provide adequate protection for the American public.

William C. Galegar

William C. Galegar
Director
Robert S. Kerr Environmental
Research Laboratory

ABSTRACT

The cost-effectiveness relationships for various agricultural and desalination alternatives for controlling salinity in irrigation return flows are developed. Selection of optimal salinity management strategies on a river basin scale is described using a four level decomposition analysis. The first level describes the cost-effectiveness of individual alternatives applicable in subbasin or irrigated valley situations. Included at this level are desalination of drainage return flows with multi-stage flash distillation (MSF), vertical tube evaporation - MSF (VTE-MSF), vapor compression - VTE-MSF (VC-VTE-MSF), electrodialysis (ED), reverse osmosis (RO), vacuum freezing - VC (VF-VC), and ion exchange (IX). Feedwater is assumed to be supplied by groundwater wells or surface diversions, whereas brine disposal may be accomplished with either injection wells or evaporation ponds. Agricultural salinity control alternatives at the first level include canal, ditch, and lateral lining, and on-farm improvements (irrigation scheduling, automated surface irrigation, sprinkler irrigation, and trickle irrigation). The second level representing the best management practices for the subbasin is defined by selecting the minimum cost policy of level 1 alternatives which reduce subbasin salinity by preselected amounts. The establishment of second level cost-effectiveness functions allow evaluation of salinity management at the river subsystem cost-effectiveness functions provide the optimal basin-wide strategies and their respective structures. A case study of the Grand Valley in western Colorado is presented to demonstrate the model.

This report was submitted in fulfillment of Grant No. R-803869 Colorado State University, under the sponsorship of the U. S. Environmental Protection Agency. This report covers the period July 15, 1975 to July 14, 1977, and was completed as of October 1, 1977.

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SECTION 1

INTRODUCTION

OBJECTIVES OF STUDY

Controlling salinity in a major river basin is a difficult task because of the mixture of diffuse and point sources of salinity. Generally, the best practicable solution lies in combining the strong features of several control measures and applying each to the conditions for which it is best suited. Salinity control technology in this regard remains to be developed since few investigations have managed to integrate the alternatives. Probably the area needing first priority is the combined use of desalting and irrigation return flow quality control. In an irrigated area, for example, traditional salinity control measures include canal and lateral linings along with improved irrigation practices such as irrigation scheduling. Nevertheless, treatment of the agricultural system does not completely alleviate local salinity problems because only the salt pickup component of salinity can be reduced. By considering desalination, a total salinity control program is possible by removing salts being transported through the irrigated system thus, creating even more than a "zero discharge" capability. Desalting should therefore be considered in not only controlling the quality of irrigation return flows, but also controlling salinity from mineralized springs, seeps or highly saline groundwater.

The objective of this study was to develop an analytical procedure for optimizing salinity control strategies in salinity affected areas by integrating desalting measures. As a case study for verification of the analysis, the main stem of the Colorado River extending from the Colorado-Utah border to its headwaters will be examined. This reach of the river includes the Grand Valley where considerable agricultural related research has either been concluded or is underway on various salinity control measures. In addition, Glenwood Springs which adds more than 300,000 tons of salt annually to the river and several anticipated important energy and urban water developments which are expected to create significant salinity increases in the river are also in this region. It is, therefore, a prime area for developing such an analysis. In this regard, the specific objectives were:

1. To identify the saline water flows in the region as to quality characteristics, flow magnitude and variation, location with regards to power and labor supplies, brine disposal, and environmental impact, and requirements for collection and conveyance;
2. To determine the costs of desalting a fraction or all of these flows to achieve a range of salinity control in the river system;
3. To formulate an analytical procedure for selecting an optimal level of desalination in an area where other methods of control could also be used. This procedure requires that the cost-effectiveness of desalination be compared with similar relationships describing the other alternatives, subject to a salinity control potential, an agricultural, urban, or industrial water development plan, and a policy for maintaining or reducing salinity concentrations basin-wide; and
4. To determine the specific data requirements, research needs, and system parameters most influential on the structure of a regional salinity control technology in order to insure reliability in its eventual implementation and provide the basis for applying these results to other salinity affected river systems.

The Upper Colorado River Basin contains vast reserves of oil shale and coal essential to the future energy needs of the nation. The rapidly growing urban centers of Denver, Salt Lake City, and Albuquerque will require substantial interbasin transfers to meet their water resource needs. These developments will compound the already serious salinity problem in the basin and steps must be taken to offset the expected damages. Because of the serious nature of the salinity problem in the Lower Colorado River Basin, the most binding constraints on future water resource developments in the Upper Basin might very well be salinity rather than each state's entitlement under the Colorado River Compact of 1922. The specific recommendations for resolving this problem are still being investigated with the exception of possibly the decision to construct the desalting facilities on the Wellton-Mohawk Drain. There has, however, been a statement of policy by the basin states and the U. S. Environmental Protection Agency to the effect that salinity concentrations should be maintained at or below existing levels. The EPA has supported salinity related investigations in the basin for a number of years to identify alternative control measures and their feasibility. The Bureau of Reclamation through a Congressional mandate has been given the responsibility of planning for and implementing salinity controls in the basin. Both agencies will necessarily rely on previous investigations in coordinating their policy and instigating other studies as new research needs become apparent. The results of this project, an evaluation which optimizes agricultural salinity control and desalting, are intended as an aid to planners in developing an effective salinity control program.

The efforts involved in completing this investigation fell quite naturally into four phases of work:

1. Developing an optimizational analysis for the problem of managing salinity in a river basin;
2. Modeling the cost-effectiveness relationships associated with alternative improvements in the irrigation system to improve flow quality;
3. Simulating the costs of desalting saline flows; and
4. Determining the least cost combination of agricultural and desalting measures which achieve a desired level of salinity control in the study area.

SECTION 2

CONCLUSIONS

Desalting is a very expensive but nevertheless feasible salinity control alternative. Unit costs are inversely proportional to plant capacity and feedwater salinity and proportional to land, energy, and material prices. It appears a reverse osmosis system having feedwater in the 7,000-10,000 mg/ℓ range will offer the most cost-effective system for a regional salinity management application. The capital costs (1976 base) for such a system are approximately $320 for a 30 year removal of one metric ton annually.

Desalting cost estimates are affected by several site-specific conditions like land costs, environmental concerns, climatic conditions, energy availability, and labor. The supply of feedwater and the removal and disposal of brines are also serious considerations in desalting system designs. In the Upper Colorado River Basin or others of a similar nature, these factors would not significantly alter the unit cost noted above.

Because irrigated agriculture is a large contributor to most western salinity problems, the feasibility of desalination depends on the cost-effectiveness of improving irrigation efficiency. The reduction of salinity concentrations in irrigation return flows is primarily a matter of minimizing the subsurface component of the hydrology by lining conveyance systems to reduce seepage, increase on-farm irrigation efficiency to diminish deep percolation, and utilize field relief drainage to intercept deep percolation. Desalting the return flows is generally a more cost-effective alternative than relief drainage and compares favorably with large canal linings in low seepage areas. However, most on-farm improvements such as head ditch linings, automation of surface irrigation, conversion to sprinkler systems, or better water management practices (irrigation scheduling) have better cost-effectiveness characteristics than desalting. Conversion to trickle irrigation and desalting return flows have comparable cost-effectiveness. It should be emphasized that the relative feasibilities of these technologies are highly dependent on site specific factors and can change measurably from location to location.

Salinity control strategies involve complex, constrained and nonlinear mathematics when reduced to their most elemental form. Trade-offs exist among alternative salinity control measures and some are prerequisite on the implementation of others. It is therefore difficult, if not impossible, to identify the best program to implement without incorporating an optimizational analysis. A case study of the Grand Valley of western Colorado using these tools demonstrated a 20-30% cost savings over the existing salinity control plan. On a larger scale such as the Upper Colorado River Basin, the benefits of optimization basin planning could amount to many millions of dollars annually.

Unfortunately, there is not enough data in most irrigated areas for a Grand Valley type investigation and one may be inclined to wonder at the value of this study. The problem in other areas will be delineating the best management practices at the first levels of salinity control planning. It should be possible to identify the relative emphasis on desalting on-farm improvements, conveyance system linings, and drainage, but not the individual characteristics of these alternatives. Consequently, the value of the optimizational techniques in the planning process will occur in two stages. The first analysis can identify the priority among the primary control measures to serve as a guide to more detailed studies. Then as is the case in the Grand Valley, the process can be repeated with the added data to determine the policies for implementation, on a more detailed level.

SECTION 3

RECOMMENDATIONS

The results of this project should be extended to the basin level development of optimal salinity control policies in the Upper Colorado River Basin, the Rio Grande River Basin, and others facing critical salinity management decisions. These studies are needed to provide agencies having the responsibility for implementing salinity control programs with information that will maximize the effectiveness of funds and manpower. The dimensionless curves distributing costs and salt loading reductions (or return flow rates) should be used in lieu of the costly investigations necessary to define such relationships in each individual subbasin or valley.

The models presented in this report have only been partially tested with respect to the sensitivity to important parameters and assumptions that were made during the formulations. This work should be completed to not only establish the reliability of the models but also to identify the field data having the most impact in determining the optimal salinity control strategies. Since these models are dependent on predictions made by more detailed hydro-salinity simulation models, the sensitivity to the simulation model assumptions and parameters should also be evaluated.

The costs of building and operating desalination systems should be updated in the form utilized in this report. Technological advances since 1972 need to be included as well as inflational cost increases not completely encompassed by various cost indexes. More efforts are also needed in evaluating the costs of irrigation system improvements.

Evaluative techniques for defining irrigation efficiencies in large areas with available or easily collected field data should be developed for the water quality planning agencies and consultants.

SECTION 4

COST-EFFECTIVENESS ANALYSIS

INTRODUCTION

There is probably no other means as commonly used or as widely accepted for evaluating the merits of a water resource system as its economic attributes. Although water resources can be classified primarily as public commodities, significant influences on pricing and management are due to water uses in the private market. In most states, water is not legally "owned" by an individual other than the state, but rights can be obtained for the use of water by individuals. However, when the legal interpretation implies that the water is tied to the land and cannot be transferred, then the value of the land is enhanced by its water right. These cases give water a market value obtainable by a right holder even when the resource is administered as public property. As in the case of grazing privileges on public lands, the pricing is usually lower than that obtainable in the private economy. As a consequence, right holders are often reluctant to accept changes to improve their use efficiency and thereby reduce their water requirement.

Reservoirs, diversion works, and distribution systems aid management of water which tends to remain fixed in spatial distribution but randomly distributed with time. These facilities, without which water use would be constrained to local utilization, allow wider water use between adjoining watersheds and along a river system. However, the diversion of waters for most uses create externalities (downstream water quality detriments, for example) which are usually not considered by local planners. Thus, maximum economic efficiencies are only achieved when the economic evaluations assume a regional interpretation.

This section deals with an optimization procedure intended to determine the most cost-effective means of managing salinity from non-point agricultural sources. The interpretation is on a regional scale so that economic efficiency is addressed.

Optimization Criterion

Optimization is generally a maximization or a minimization of concise numerical quantities reflecting the relative importance of the goals and purposes associated with alternative decisions. Of themselves, neither the goals nor purposes directly yield the precise quantitative statements required by systems analysis procedures. Therefore, the objectives require a mathematical description before alternative strategies can be evaluated (Hall and Dracup, 1970). Presumably, such a comparison would permit a ranking of these policies as a basis for decision making. The specific measure to facilitate this examination can be defined as the optimizing criterion.

The central problem is to link the descriptions of the physical environment via mathematical models with the social and political environment (Thomann, 1972). Probably the most commonly used and widely accepted "indicators" are found among the many economic objective functions. However, considerable controversy exists as to the most realistic of these tools. If, for example, aspects of a water quality problem could be priced in an idealized free market monetary exchange, the forces that operated would insure that every individual's marginal costs equalled his marginal gains, thereby insuring maximum economic efficiency. In the absence of this ideal situation water quality cannot be quantified with a high degree of accuracy and the optimizing criterion in any case is at best an indicator of the particular alternative.

Among the more adaptable economic indicators are maximization of net benefits, minimum costs, maintaining the economy, and economic development. The use of each depends on the ability to adequately define tangible and intangible direct or indirect costs and benefits. In water resource development, and water quality management specifically, the economic incentives for more effective resource utilization are negative in nature (Kneese, 1964). A large part of this problem stems from the fact that water pollution is a cost passed on by the polluter to the downstream user. Consequently, the inability of the existing economic systems to adequately value costs and benefits has resulted in the establishment of water quality standards, however, inefficient these may be economically (Hall and Dracup, 1970). The immediate objective of water resource planners is thus to devise and analyze the alternatives for achieving these quality restrictions at minimum cost, the criteria chosen for this study.

OPTIMIZATION METHOD

The search for an optimizing technique to evaluate the relative merits of an array of alternatives depends largely upon the form of the problem and its constraints. While the allegorical Chinese maxim cited by Wilde and Beightler (1967) stating "There are many paths to the top of the mountain, but the view there is always the same," is also true in this case; not every method can be applied with the same ease. Each optimization scheme has its unique properties making it adaptable to specific problems, although many techniques when sufficiently understood can be modified to extend their applicability.

Most conditions encountered in irrigated agriculture involve mathematical formulations which are nonlinear in both the objective function and the constraints. Furthermore, the constraining functions may be mixtures of linear and nonlinear equalities and inequalities. Without simplifying these problems or radically changing existing optimization techniques, it is possible to derive solutions based upon what Wilde and Beightler (1967) describe as the "differential approach."

Most techniques for selecting the optimal policy do so by successively improving a previous estimate until no betterment is possible. These may be classified as direct or indirect methods depending on whether they start at a feasible point and stepwise move toward the optimum or solve a set of equations which contain the optimum as a root. In a majority of cases, the differential approach can be used to describe the method.

The optimizing technique used in this effort is called the "Jacobian Differential Algorithm." Theoretically, it is a generalized elimination procedure which is computationally feasible under a wide variety of conditions. The characteristics of convexity are assumed and since the maximization problem is simply the negative of a minimization one, the following discussion will be limited to the latter case. As in all direct minimizing procedures, the algorithm involves four steps:

1. Evaluate a first feasible solution, $\underline{x}^o$, which satisfies the problem constraints. The underbar indicates vector notation and the superscript ^{o} is used to describe the "old" or initial points;
2. Determine the direction in which to move such that the objective function, y, is decreased most rapidly. This requires a move from $\underline{x}^o$ to the new point, $\underline{x}^\nu$, in which the superscript $^\nu$ represents the new point notation;
3. Find the distance that can be moved without violating any of the problem constraints; and
4. Stop when the optimum is reached.

The user is left only with providing the first feasible solution, step 1. This may seem to be a drawback for the problem, but in real situations a feasible solution already exists as a current policy. Step 4 is accomplished by an examination of what are now referred to as the "Kuhn-Tucker conditions." These criteria do not indicate whether the procedure has reached a local or global optimum; consequently, it is necessary to derive a means for checking.

Theoretical Development

Consider the problem in which the minimum value of the objective function is sought subject to a set of constraining functions. Writing this problem mathematically,

$$y = \min y(\underline{x}) \quad \text{......} \quad (1)$$

subject to,

$$\underline{f}(\underline{x}) \geq \underline{0} \quad \text{......} \quad (2)$$

where the notation $y(\underline{x})$ denotes "as a function of the vector x." The number of x variables is defined as N and the number of constraints as K. The method of analysis depends largely upon the structure of the constraints. When all the constraints are inequalities and "loose" or "inactive" (strictly >) at the initial feasible point $\underline{x}^o$, the problem is "unconstrained." In the other case when either some of these functions are strict equalities or when some of the inequalities are "tight" or "active," the problem is referred to as "constrained." Although both of the conditions may occur in the solution of a problem, they require somewhat different approaches as the algorithm progresses toward the optimum.

Elimination Procedure --

The elimination nature of the technique is derived from the fact that it is at least conceptually possible to employ only the currently active constraints to eliminate some of the x's from the problem, making it temporarily unconstrained. To begin, define the number of active constraints as T and reorder the constraint set so that the first T is the active constraint with index t = 1, 2, ..., T. Further, introduce "slack" variables to the active constraints so they take the form,

$$\underline{f}(\underline{x}) - \underline{\phi} = \underline{0} \quad \text{......} \quad (3)$$

and become strict equalities, where $\underline{\phi}$ is the vector of slack variables. The purpose of this transformation is that by continual observation of the slack variables the distinction between active and inactive functions can be determined. The problem now contains N original variables plus T slack variables which are related by T active constraints. If the constraints are linear, T of the variables can be eliminated from the objective function by the constraint expressions, making the problem unconstrained. However, in the general situation, the constraints are nonlinear, and it is not directly possible to substitute for the dependent variables, but rather to first linearize the functions by taking the first partial derivatives with respect to the x variables. Even though the nonlinearity may still exist due to the nature of the terms in the constraints, if it is assumed that the changes toward the optimum point are sufficiently small, then only a small deviation is introduced. The elimination procedure takes place by partitioning the variable set into "states" and "decisions." The state variables are the selected variables which are to be eliminated by the T active constraints. The decision variables are the remaining independent variables which will be employed to seek the minimum value of the objective function. The criteria for the partition include two aspects:

1. All slack variables are taken as decisions unless no other x-variable is available to be a state variable. Since all ϕ_t are identically equal to zero, when the algorithm moves from the old point $\underline{x}^o$ to the new one $\underline{x}^\nu$ in its search for the minimum there is a 50 percent chance that the ϕ_t will become negative. This is a violation of the problem constraints; and
2. Since the same basic reasoning applies to the x-variables, the largest absolute valued variables are best suited to be state variables.

In the computer code of the algorithm described in Appendix B, the selection of states and decisions is undertaken in a much more complex procedure to insure numerical stability.

After partitioning the x-vector into state and decision variables, the variables can be relabeled s for states and d for decisions. Equation 1 at the initial point $\underline{x}^o$ can then be written,

$$y = \min y(s_1, s_2, \ldots, s_T, d_1, d_2, \ldots, d_D) \qquad (4)$$

in which D is the number of decision variables and equals (N - T). In addition, the constraints listed in Eq. 3 can be rewritten as:

$$\underline{f}(\underline{s}, \underline{d}) - \underline{\phi} = \underline{0} \qquad (5)$$

The next step is to employ the chain rule of calculating the total differential of y. In vector notation,

$$\partial y = (\nabla_s y)\, \partial \underline{s} + (\nabla_d y)\, \partial \underline{d} \qquad (6)$$

where the symbol ∂y is used to denote the total differential rather than the standard notation of dy. This modification is made so that the d can be reserved to denote the decision variables.

The derivatives of the constraining functions can also be written in vector form,

$$(\nabla_s \underline{f})\, \partial \underline{s} + (\nabla_d \underline{f})\, \partial \underline{d} - \partial \underline{\phi} = \underline{0} \qquad (7)$$

where the gradient, ($\nabla_s \underline{f}$), is called the Jacobian Matrix, $\underline{J}$, and the matrix ($\nabla_d \underline{f}$) can be relabeled as $\underline{C}$. Employing these variables in Eq.7 and rearranging terms:

$$\underline{J}\partial\underline{s} = -\underline{C}\partial\underline{d} + \partial\underline{\phi} \tag{8}$$

The vector $\partial\underline{s}$ can be solved for if the Jacobian matrix is always taken non-singular:

$$\partial\underline{s} = -\underline{J}^{-1}\underline{C}\partial\underline{d} + \underline{J}^{-1}\partial\underline{\phi} \tag{9}$$

The elimination of the states is now possible by substitution of Eq. 9 into Eq. 6. After rearranging terms, the final unconstrained equation is developed:

$$\partial y = \left[\nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C}\right] \underline{\partial d} + (\nabla_s y)\underline{J}^{-1}\partial\underline{\phi} \tag{10}$$

Kuhn-Tucker Conditions --

By definition of the total differential, another expression can be written in terms of the variables indicated in Eq. 10. If the elimination of the state differentials were accomplished then the total differential of y would be written:

$$\partial y = \frac{\delta y}{\delta \underline{d}}\ \partial\underline{d} + \frac{\delta Y}{\delta \phi}\ \partial\phi \tag{11}$$

in which $\delta y/\delta\underline{d}$ and $\delta y/\delta\phi$ are called "constrained derivatives." The deviation in notation is made to distinguish the $\partial y/\partial\underline{x}$, which is a partial derivative viewing all variables as independent, from $\delta y/\delta\underline{d}$ which is a partial derivative considering T of the variables as functions of the remaining N variables. By comparing Eqs. 10 and 11 it can be seen that,

$$\frac{\delta y}{\delta d} = \nabla_d y - (\nabla_s y)\underline{J}^{-1}\underline{C} \tag{12}$$

and,

$$\frac{\delta y}{\delta \phi} = (\nabla_s y)\underline{J}^{-1} \tag{13}$$

The solution of Eqs. 12 and 13 when equated to zero yield a stationary point when the decision variables are free, or in other words, allowed to assume any positive or negative value. In most instances, decision variables are not free, but subject to non-negativity conditions. Stationary points may be local or global minimums, maximums, or inflection points. The evaluation of stationary points in these cases will depend on criteria reported by Kuhn and Tucker (1951) which provide necessary and sufficient conditions for a minimum. In the problem solution at the feasible point under examination, a minimum exists if the following conditions are met:

1. Necessary conditions prerequisite for a minimum must consist of the following:

$$\frac{\delta y}{\delta d_j} \geq 0,\ d_j \leq 0, \text{ and } \frac{\delta y}{\delta d_j} d_j = 0, j = 1,2,\ldots,D \tag{14}$$

and

$$\frac{\delta y}{\delta \phi} \geq 0,\ \phi_t \geq 0, \text{ and } \frac{\delta y}{\delta \phi_t} \phi_t = 0,\ t = 1,2,\ldots,T \tag{15}$$

2. If Eqs. 14 and 15 are satisfied, then sufficient conditions for a minimum are:

$$\frac{\delta y}{\delta d_j} > 0 \ j = 1,2,\ldots,D \tag{16}$$

and

$$\frac{\delta d}{\delta \phi_t} > 0 \ t = 1,2,\ldots,T \tag{17}$$

The minimum has been reached when both the necessary and sufficient conditions have been satisfied. However, if for example, $\delta y/\delta d_j$ equals zero and $d_j \geq 0$, the tests are inconclusive since the sufficient conditions have not been met. In this case, it is necessary to take the second derivatives of the objective function with respect to the x-vector. This analysis yields a square matrix of second order partial derivatives called the Hessian matrix written mathematically as:

$$\underline{H} = \nabla_x^2 y \tag{18}$$

In order for the stationary point to be a minimum (local or global) the value of the Hessian matrix must be positive-definite, and since the properties of positive-definite matrices can be found in most texts on linear algebra, no further description will be given here.

Evaluation of Optimal Direction --

In addition to the description of the fundamental elimination technique of this optimizing technique, the preceding sections also provided the definition of the constrained derivatives of the objective function in terms of the decision and slack variables. Furthermore, criteria were given with which these parameters can also be evaluated to see when the minimum is achieved. In this section, these same derivatives will be used to determine the direction a particular decision variable, d_p or ϕ_p, must be "moved" in order to create the maximum reduction in the value of the objective function during each iterative step. Among the nonlinear programming techniques for optimization, several essentially alter all of the decision variables at each iteration. In the Jacobian

Differential Algorithm, one decision variable (d_p or ϕ_p) is selected from among the set which when moved will result in the most progress toward the minimum. If an individual term from Eq. 11 is written in discrete element form, the new value of the decision variable (or slack variable) can be determined,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta d_i}\right)^{o} \left(d_i^{\nu} - d_i^{o}\right) \quad \ldots\ldots\ldots\ldots (19)$$

or,

$$y^{\nu} - y^{o} = \left(\frac{\delta y}{\delta \phi_t}\right)^{o} \phi_t^{\nu} \quad \ldots\ldots\ldots\ldots (20)$$

where the reader is reminded that the superscripts ^{o} and $^{\nu}$ refer to the functional evaluations made at the old and new feasible solutions. It may also be worth mentioning that ϕp can only be increased, whereas d_p can be also decreased (assuming the non-negativity constraints are not violated). As a result, the increase in a slack variable is in reality a loosening of an active constraint.

The choice of the decision variable or the slack variable to be modified is primarily made on the basis of largest absolute value among the respective constrained derivatives. Three general categories are examined. To begin with, the largest positive valued derivative with which the associated decision variable is greater than zero is determined and the Kuhn-Tucker conditions are checked according to the previous section. Mathematically, this first alternative can be written,

$$\text{find: max} \left[\frac{\delta y}{\delta d_i} > 0 \,\middle|\, d_i > 0,\ i = 1,2,\ldots,D\right] \quad \ldots\ldots\ldots\ldots (21)$$

where the notation $\big| d_i > 0$ means "subject to the value of d_i being positive."

The second alternative selection for the step direction is in the negative constrained derivatives. In this case, the specific decision variable will be increased and unless an upper bound on the variable is imposed, no examination of the decision need be made. Symbolically then,

$$\text{find: min} \left[\frac{\delta y}{\delta d_i} \geq 0,\ i = 1,2,\ldots,D\right] \quad \ldots\ldots\ldots\ldots (22)$$

Finally, the largest reduction in the objective function may be facilitated by loosening a particular active constraint. Unless the constrained derivative of y with respect to the slack variable is negative, the Kuhn-Tucker conditions are satisfied. Therefore, this solution can be expressed as:

$$\text{find: } \min \left[\frac{\delta y}{\delta \phi_t} > 0,\ t = 1,2,\ldots, T \right] \ldots\ldots\ldots\ldots\ldots\ldots (23)$$

Once these maximums and minimums have been selected, the next item is to compare them with each other and select the largest absolute valued one. After having made the choice, the index on the specified decision or slack variable is now denoted by a "p", and these variables now become d_p or ϕp depending on the decision among alternatives.

Determining the Step Size --

Because the particular decision variable or slack variable to be modified has been selected, the remaining decisions and slacks will remain constant and can therefore be temporarily ignored. The next computation necessary is to determine which of the boundaries of the problem are approached first. If the non-negativity constraints on the variables are in effect, one consideration is how far a decision or slack variable can be moved without forcing a state variable to become negative. In order to accomplish this, the constrained derivatives of each state variable with respect to the particular decision or slack variable are computed using Cramer's rule on the matrix of system derivatives. From these values, the maximum move may be computed. Writing the appropriate relationships in discrete form,

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta d_p}\right)^{o} \left(d_p^{\nu} - d_p^{o}\right) \ldots\ldots\ldots\ldots\ldots\ldots (24)$$

or for the slack variables:

$$(s_i^{\nu} - s_i^{o}) = \left(\frac{\delta s_i}{\delta \phi_p}\right)^{o} \phi p^{\nu} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

Three cases exist in which a state variable can be driven to zero, namely a decrease in d_p, an increase in d_p, and an increase (or loosening) in ϕ_p. Since a search is necessary among the state variables to see which specific state goes to zero first, Eqs. 24 and 25 can be incorporated:

Case 1. Decreasing d_p

$$d_p^{\nu} = \max \left[d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} > 0 \right] \ldots\ldots\ldots\ldots\ldots\ldots (26)$$

Case 2. Increasing d_p

$$d_p^{\nu} = \min \left[d_p^{o} - \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta d_p} < 0 \right] \ldots\ldots\ldots\ldots\ldots (27)$$

Case 3. Increasing ϕ_p.

$$\phi_p^{\nu} = \min \left[- \frac{s_i^{o}}{\left(\frac{\delta s_i}{\delta \phi_p}\right)^{o}} \;\middle|\; \frac{\delta s_i}{\delta \phi_p} < 0 \right] \ldots\ldots\ldots\ldots\ldots (28)$$

The next possible limitation on the change in the decision or slack variables is the forcing of a previously inactive constraint into an active role in the problem. In order to facilitate this analysis, the constrained derivatives of the loose slack variables is computed. Again, three conditions must be considered:

Case 1. Decreasing d_p

$$d_p^{\nu} = \max \left[d_p^{o} - \frac{(\phi_\ell^{+})^{o}}{\left(\frac{\delta f_\ell^{+}}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta f_\ell^{+}}{\delta d_p} > 0 \right] \ldots\ldots\ldots\ldots\ldots (29)$$

Case 2. Increasing d_p

$$d_p^{\nu} = \min \left[d_p^{o} - \frac{(\phi_\ell^{+})^{o}}{\left(\frac{\delta f_\ell^{+}}{\delta d_p}\right)^{o}} \;\middle|\; \frac{\delta f_\ell^{+}}{\delta d_p} < 0 \right] \ldots\ldots\ldots\ldots (30)$$

Case 3. Increasing ϕ_p

$$\phi_p^{\nu} = \min \left[- \frac{(\phi_\ell^{+})^{o}}{\left(\frac{\delta f_\ell^{+}}{\delta \phi_p}\right)^{o}} \;\middle|\; \frac{\delta f_\ell^{+}}{\delta \phi_p} < 0 \right] \ldots\ldots\ldots\ldots (31)$$

A final limitation which should be noted is when a decrease in d_p is to be made and neither condition above is violated before non-negativity is encountered. In such a case, the maximum decrease would be $- d_p$ assuming the non-negativity conditions hold. Once this and the other values of d_p and ϕ_p have been made, the most limiting case is evaluated.

If by varying a slack or decision variable a state is driven to zero, a decision variable must be selected to trade positions with the state to avoid zero valued state variables. In addition, when a loose constraint is tightened, a new state variable must be selected from the rest of the decision variables. The exception to this is when a loose constraint is tightened by loosening a currently active constraint.

APPLICATION TO REGIONAL SALINITY PROBLEMS

On a basin-wide scale, a salinity problem is the combined effect of many irrigated areas, saline springs, diffuse natural inflows, and other miscellaneous sources. These salinity sources not only occur sequentially due to the geographic structure of an hydrologic area, but are also often governed by differing administrative formulas. Consequently, the problem of determining an "optimal" strategy for a large area like the river basin rapidly becomes too large and too complex for direct analysis. One of the various mathematical techniques for optimizing complicated systems is to decompose the problem into a series of subproblems whose solutions are coordinated in a manner that produces the solution to the larger problem. One method applied to analysis of water quality improvements in the Utah Lake Drainage basin of central Utah provides both a simple and effective decomposition (Walker, et al. 1973). The structure of the decomposition methodology referred to above is shown schematically in Figure 1. Individual levels of modeling are delineated to define water quality cost-effectiveness analyses at different stages of development enroute to a single representation at the ultimate basin-wide scale.

Conceptual Salinity Control Model

The conceptual model illustrated in Figure 1 represents an additive approach for determining the minimal cost salinity control strategy in a river basin. A number of levels or subdivisions having similar characteristics can be defined to correspond to various levels of hydrologic or administrative boundaries in a region. Within each level, the alternative measures for salinity management are characterized by cost-effectiveness relationships. A more detailed review of the structure of cost-effectiveness functions and their interdependence will assist the reader in understanding the application of the conceptual model in later sections.

Description of Cost-Effectiveness Functions --

The alternatives for managing salinty on a basin-wide scale fall into two categories: (1) those that reduce salinity concentrations by dilution or minimizing the loss of pure water from the system by evaporation; and (2) those that improve water quality by reducing the mass emission of salt.

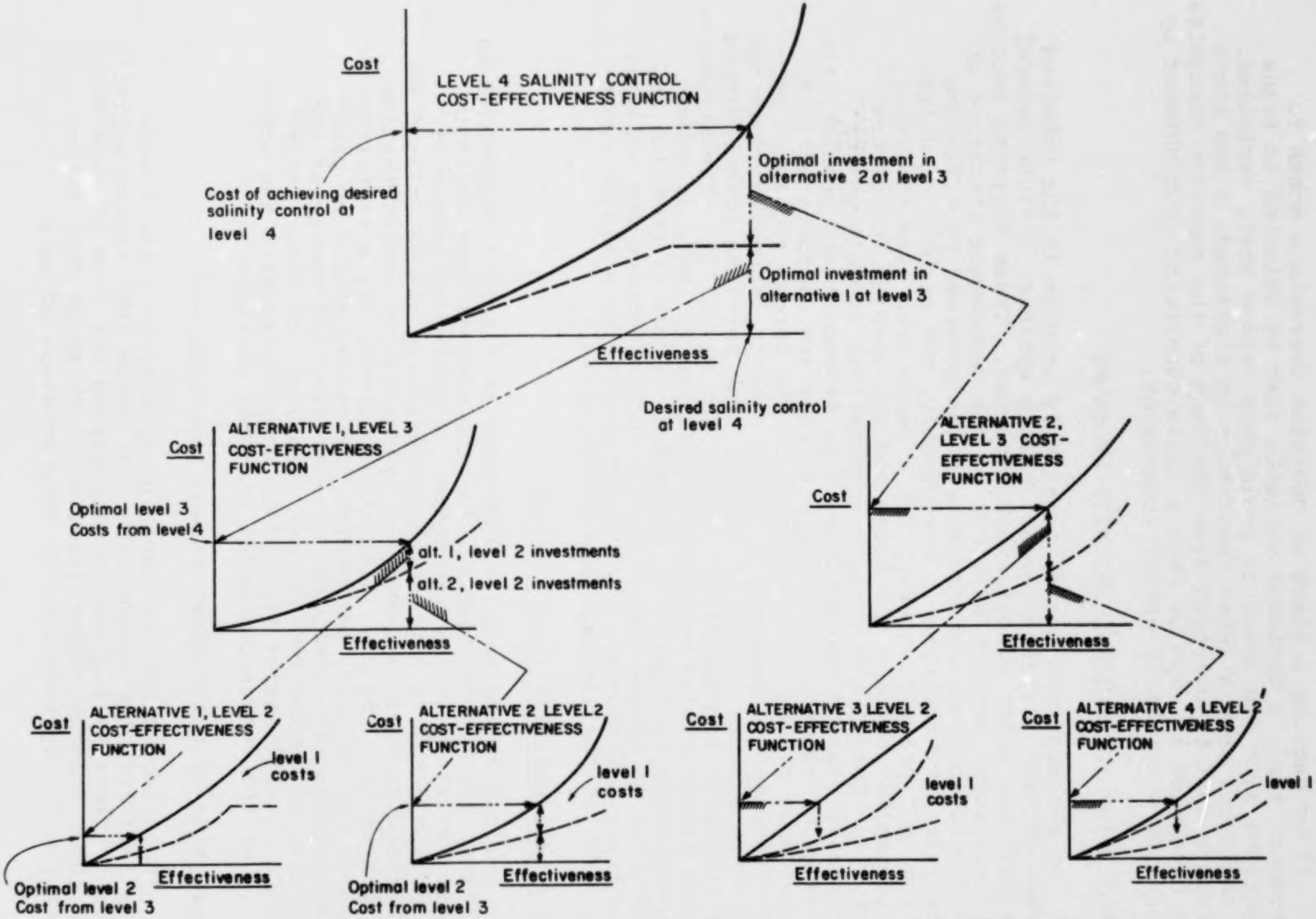

Cost
LEVEL 4 SALINITY CONTROL COST-EFFECTIVENESS FUNCTION
Cost of achieving desired salinity control at level 4
Optimal investment in alternative 2 at level 3
Optimal investment in alternative I at level 3
Effectiveness
Desired salinity control at level 4
ALTERNATIVE I, LEVEL 3 COST-EFFCTIVENESS FUNCTION
Cost
Optimal level 3 Costs from level 4
alt. I, level 2 investments
alt. 2, level 2 investments
Effectiveness
ALTERNATIVE 2, LEVEL 3 COST-EFFECTIVENESS FUNCTION
Cost
Effectiveness
ALTERNATIVE 1, LEVEL 2 COST-EFFECTIVENESS FUNCTION
Cost
level I costs
Effectiveness
Optimal level 2 Cost from level 3
ALTERNATIVE 2 LEVEL 2 COST-EFFECTIVENESS FUNCTION
Cost
level I costs
Effectiveness
Optimal level 2 Cost from level 3
ALTERNATIVE 3 LEVEL 2 COST-EFFECTIVENESS FUNCTION
Cost
level I costs
Effectiveness
ALTERNATIVE 4 LEVEL 2 COST-EFFECTIVENESS FUNCTION
Cost
level I
Effectiveness

Figure 1. Conceptual decomposition model of a regional or basin salinity control strategy.

Examples of the first category include weather modification to enhance stream flow, evaporation suppression, and phreatophyte control. Many of these approaches are more costly and difficult to apply than is justified by the salinity control achieved and are therefore not considered in this work. In the second category, such measures as saline flow collection and treatment, reduction in agricultural return flows, and land use regulation can be used to reduce the volume of salinity entering receiving waters. In this report, only saline flow collection and treatment and irrigation return flow management are evaluated. Under these assumptions, salinity control becomes a mutually exclusive problem that allows addition of individual solutions to derive larger solutions. By letting the spatial scale of the problem correspond to successive layering or additions, the multilevel approach is congruent to the subbasin breakdown of major hydrologic areas.

The smallest spatial scale considered in this analysis is that of a subbasin containing an irrigated valley or stream segment delineated by inflow-outflow data. In a major river basin, a number of river systems may combine to form the basin itself so there are actually three subdivisions in a river basin. Thus, vertical integration of subbasins yields river subsystems and integration of river subsystems yield the aggregate river basin. In this analysis the river basin, river subsystem, and subbasin divisions have been designated as levels 4, 3, and 2, respectively. Level 1 will also encompass the subbasin scale as will be described shortly.

Associated with each level of the model are cost-effectiveness functions describing each alternative for controlling salinity. The structure of the cost-effectiveness functions includes two parts. The first is the function itself. In order to compare the respective feasibility among various salinity control measures at each level, the mathematical description of each alternative must be in the same format. Since this study involves evaluating the minimal cost strategy for reducing salt loading, each salinity control measure's feasibility for being included in the eventual strategy is based on the relationship between the costs of improvement and the resulting reduction in salt loading. The second part of the cost-effectiveness functions is what might be called a "policy space". To appreciate this aspect of the model it is probably necessary to first discuss the determination of the optimal basin-wide strategy.

Evaluating the Optimal Strategy--

Suppose the optimal policy for controlling salinity in a river basin had been determined with a minimum cost decision criterion. Such an analysis would provide two pieces of information. First, it would detail the cost associated with a range of reductions in salinity, and second, it would delineate how much of these costs are to be expended in each river subsystem. In other words, the evaluation of the optimal strategy at level 4 involves systematic comparisons of level 3 cost-effectiveness functions and once the strategy had been determined, it also yields the optimal costs or expenditures in each level 3 alternative (river subsystem). In a similar vein, the level 2 costs and policies are determined from a knowledge of the level 3 optimal as determined during the level 4 analysis, and so on. Thus, the cost-effectiveness function for any alternative within a level is:

1. the result of optimization of respective cost-effectiveness functions at a lower level and therefore a minimum cost relationship at every point; and
2. the sum of costs from optimal investments into each alternative at a lower level. The "policy space" is therefore a delineation of lower level cost-effectiveness function.

The preceding paragraphs noted the detailing of salinity control strategy once the optimal is known. Determining the basin optimal, on the other hand, begins at level 1. A comparison of level 1 cost-effectiveness functions describing each alternative at that level produces the array of level 2 functions. Similar steps yield each succeeding level's optimal program. Thus, the multilevel approach described herein involves a vertical integration up through the levels to determine the optimal policy and a backwards trace to delineate its components.

Mathematical Salinity Control Model

Consider a single salinity control measure within a subbasin such that its cost-effectiveness characteristic can be written:

$$y_i^1 = f(x_i^1) \quad \text{.....................................} \quad (32)$$

in which,

y_i^1 = total cost attributable to the ith control measure at the first level of optimization; and

x_i^1 = annual salt loading decrease associated with an expenditure of y_i^1 dollars on the ith salinity control measure.

Superscripts will refer to model level whereas subscripts will designate alternatives. It is assumed that the relationship between y_i^1 and x_i^1 can be determined and that the total potential reduction in salt loading for the ith measure at level 1 is X_i^1.

The optimal salinity control strategy in a subbasin is the minimum cost array of individual measures (y_i^1) which achieve the desired degree of salinity control. The optimum may be determined as:

$$y_j^2 = \min \sum_{i=1}^{n} y_i^1 \tag{33}$$

subject to,

$$x_i^1 \leq X_i^1 \;;\; i = 1,2,\ldots,n \tag{34}$$

$$\sum_{i=1}^{n} x_i^1 = x_j^2 \tag{35}$$

where,

x_j^2 = salt load reductions targeted for the jth subbasin at the second level;

y_j^2 = minimum cost of reducing x_j^2 tons of salt from the subbasin; and

n = number of individual salinity control measures per subbasin.

If Eqs. 33, 34, and 35 are solved repeatedly for values of x_j^2 ranging up to the maximum value attainable in the subbasin, X_j^2, then a cost-effectiveness relationship between y_j^2 and x_j^2 can be determined:

$$y_j^2 = f(x_j^2) \tag{36}$$

Similar analysis for all other subbasins yields a family of second level cost-effectiveness functions.

For each river subsytem, the preceding analysis is repeated to determine a family of cost-effectiveness curves for level 3. Specifically,

$$y_k^3 = \min \sum_{j=1}^{m} y_j^2 \tag{37}$$

subject to,

$$x_j^2 \leq X_j^2 \;;\; j=1,2,\ldots,m \tag{38}$$

$$\sum_{j=1}^{n} x_j^2 = x_k^3 \tag{39}$$

in which,

x_k^3 = total annual salt load reduction for the kth river subsystem (level 3);

y_k^3 = total costs of reducing x_k^3 tons of salt from the subsystem; and

m = number of subbasins per river subsystem.

Again, solution of the subsystem analysis for the range of possible salt reductions, $0 \leq x_k^3 \leq X_k^3$, yields the relationship:

$$y_k^3 = f(x_k^3) \quad \text{...(40)}$$

At the final level, level 4, corresponding to the solution of the salinity control strategy at the river basin scale,

$$Y = \min \sum_{k=1}^{\ell} y_k^3 \quad \text{.................................(41)}$$

subject to,

$$x_k^3 \leq X_k^3 \ ; \ k=1,2,\ldots,\ \ell \quad \text{.................................(42)}$$

$$\sum_{k=1}^{\ell} x_k^3 = X_T \quad \text{...(43)}$$

where,

X_T = total annual reduction in salt load expected in the basin;

Y = total costs in achieving a X_T reduction; and

ℓ = number of river subsystems in the basin.

By varying X_T over its possible range and obtaining the basin-wide cost-effectiveness relationship,

$$Y = f(X_T) \quad \text{...(44)}$$

planners and regulatory agencies have information they can utilize in deciding what degree of salinity control to implement, where to implement it, and what measures to employ.

SECTION 5

SIMULATION OF DESALTING COST-EFFECTIVENESS

INTRODUCTION

The development of desalination technology in the United States has been guided by the basic objective outlined by Congress to the U. S. Department of the Interior's Office of Saline Water (now combined with the Office of Water Resources Research to form a single department entitled, "Office of Water Research and Technology"). This objective is:

> "to provide for the development of practicable low-cost means for producing from sea water or from other saline waters (brackish and other mineralized or chemically charged waters), water of a quality suitable for agriculture, industrial, municipal, and other beneficial consumptive uses."

The objective of desalination as listed above has been given a massive research and development effort although the application to large scale systems is only now beginning to occur. The traditional scope of saline water conversion programs has been to reclaim otherwise unsuitable waters for specific needs. However, this scope has dealt almost exclusively with utilization of product water directly rather than returning it to receiving waters in order to improve the overall resource quality. Thus, with mounting concerns for managing salinity on a regional or basin-wide scale, the potential for applying desalination within the framework of an overall salinity control strategy is an interesting one. In fact, the use of desalting systems to resolve critical salinity problems is already being planned as part of the Colorado River International Salinity Control Project agreement between the United States and the Republic of Mexico (U. S. Department of the Interior, 1973).

In the context of regional salinity control, desalting costs can be expressed in dollars per unit volume of salt extracted in the brine discharge rather than the conventional index of costs per unit volume of reclaimed product water. In this manner the respective feasibility of desalination and other alternatives for salinity management can be systematically compared during the processes of developing strategies for actual implemenatation of salinity controls. A desalting system as used herein consists of facilities for supplying raw water (water to be desalted) to the plant, the desalting plant itself, and facilities to convey and dispose of the brine. Transportation of product water beyond the confines of this system is not considered.

The cost simulations described in this section are intended to represent the "reconnaissance level" sensitivity to cost estimating input parameters and are not, therefore, inclusive of the many factors necessary for detailed "definite plan" level estimates. For example, internal design optimization, alternative equipment from various manufactures, and many climatic, environmental, or topographic conditions are not included. At this level of sophistication, two major references have been written from which the bulk of information necessary for desalting cost simulation have been abstracted. Prehn, et al. (1970) summarized a desalting cost calculation procedure for several desalting methods and related facilities. This work was subsequently improved and expanded by the Bureau of Reclamation (U. S. Department of the Interior, 1972). These costing procedures were first mathematically simulated and then programmed for a digital computer (Appendix B), and include the following seven processing systems:

(1) multi-stage flash distillation (MSF);
(2) vertical tube evaporation - multi-stage flash distillation (VTE-MSF);
(3) vapor compression - vertical tube evaporation - multi-stage flash distillation (VC-VTE-MSF);
(4) electrodialysis (ED);
(5) reverse osmosis (RO);
(6) vacuum freezing - vapor compression (VF-VC); and
(7) ion exchange (IX).

The computer code simulation of the desalting costs for the seven processes is the end result of the desalination cost analysis described in this section and is hereafter denoted as the desalting submodel.

DESALINATION COST ANALYSIS

In general, the costs associated with desalting systems may be classified as either those expended during construction or those required annually to operate and maintain the facilities. These costs are subject to inflational pressures and must therefore be periodically updated. Once costs are current, various relationships between the costs and system performance can be formulated. A detailed description of the costing models will be presented in a later section.

Capital Costs

Construction costs, capital costs, or investment costs include all expenses associated with building the appropriate facilities and can be subdivided into the following eight categories:

(1) Construction costs, including designs and specifications, labor, and materials;
(2) Steam generation equipment if utilized in the desalting process;
(3) Site development expenses for offices, shops, laboratories, storage rooms, etc., and for the improvement of the surrounding landscape such as parking, grading, and fencing;
(4) Interest during construction on funds borrowed to finance construction;
(5) Start-up costs necessary to test the plant operation, train operating personnel, and establish operating criteria;
(6) Owner's general expenses for indirect costs like project investigation, land acquisition, contract negotiation and administration, and other miscellaneous overhead costs;
(7) Land costs for the site and conveyance facilities; and
(8) Working capital to cover daily expenses involved in plant operation.

The capital costs delineated above are based on estimating functions current during mid-1971 and therefore must be updated to prevailing price levels. The costs for construction, steam supply, and general site development can be estimated in the present time frame by employing the Engineering News Record Construction Cost Index, ENR (Engineering News Record is published monthly by McGraw-Hill, Inc.). In July of 1971 the ENR index was 952, whereas in January 1976 it had risen to 1354. Consequently, a early 1976 cost estimate would be 1354/952 or 1.42 times the 1971 functional estimate. Other capital costs for interest during construction, owners' general expense, start-up, and working capital are functions of construction, steam, and site development costs and are therefore updated automatically. Land costs may be estimated on a current basis by utilizing existing land prices.

Estimates of capital costs for feedwater and brine facilities require several other inflational factors. For example, conveyance pipeline costs are modified by the Bureau of Reclamation Concrete Pipeline Cost Index, CPI (1.17 for July 1971 and 2.13 in January 1976), and the USBR Canal and Earthwork Cost Index, EI (1.27 in July 1971 and 1.92 in January 1976).

The annualization of the construction costs is divided according to whether or not the costs represent depreciating capital. Depreciating capital costs (item 1-6 above) are multiplied by a "fixed charge factor," FCR, which is the percentage of total depreciating capital cost that is encompassed by interest, amortization, insurance, and taxes. Non-depreciating capital costs (7 and 8) are multiplied by the prevailing interest rate selected for project evaluation or incurred in borrowing.

Annual Costs

Annual operation and maintenance costs have been divided into six categories described as:

(1) Labor and materials for plant or support facility operation;
(2) Chemicals for pretreatment and process additions;
(3) Fuel to power steam generation equipment;
(4) Electricity for pumps, filters, etc.;
(5) Steam generator operation (O) and maintenance (M); and
(6) Replacement of process elements.

These annual costs must also be updated for price increases due to inflation. Labor and materials and steam generation O & M are updated using the Bureau of Labor Statistics Labor Cost Index, SIC 494-7, which was 3.76 in July 1971 and 4.93 in January 1976. Chemical costs are multiplied by the present to 1971 ratio of the Bureau of Labor Statistics cost index for chemicals and allied products (181 for 1976/104.4 for 1971). Fuel and electricity costs are estimated using present prices for these inputs and are therefore always estimated currently. Replacement costs are expressed on functions of plant capacity and are also not updated by cost indexing.

Water and Salt Costs

After describing the individual costs associated with desalting systems, it is generally necessary to express such costs in either dollars per unit volume of product water (for water supply feasibility) or dollars per unit of salt extracted (for salinity control studies). These cost bases are determined in this study by dividing the total annual costs by the annual volume of product water or brine salts.

Depreciating capital costs for the plant itself and the feedwater-brine disposal systems are multiplied by the fixed charge factor. Non-depreciating costs are next multiplied by the interest rate, added to the annualized depreciating costs, and finally summed with the remaining annual cost. Thus, using the eight capital cost categories and six annual cost elements,

$$C_{pw} = \frac{FCR \sum_{i=1}^{6} C'_i + I_r \sum_{i=7}^{8} C'_i + \sum_{i=9}^{14} C'_i}{C_p x\ U_f \times 3.65x10^{-4}} \quad \ldots\ldots\ldots\ldots (45)$$

$$C_s = \frac{FCR \sum_{i=1}^{6} C'_i + I_r \sum_{i=7}^{8} C'_i + \sum_{i=9}^{14} C'_i}{C_b \times C_{bo} \times U_f \times 3.65x10^{-4}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (46)$$

in which,

C_{pw} = unit cost of product water, \$/m^3;
C_s = unit cost of brine salts, \$/metric ton;
C'_i = total annual cost for element i, \$/year (feedwater + plant + brine) (C'_1 = construction cost, C'_2 = steam generation, C'_3 = site development, C'_4 = interest during construction, C'_5 = start-up costs, C'_6 owner's general expense, C'_7 = land, C'_8 = working capital, C'_9 = labor and materials, C'_{10} = chemicals, C'_{11} = fuel, C'_{12} = electrical, C'_{13} = steam generation O & M, and, C'_{14} = replacement.)
C_p = product water volume, m^3/day;
C_b = brine volume, m^3/day;
C_{bo} = TDS concentration in brine, mg/ℓ;
FCR = fixed charge rate;
I_r = interest rate; and
U_f = use factor - fraction of total time in actual operation.

The results from the desalting cost model indicate that unit costs as described in Eqs. 45 and 46 for either water or salt production are highly affected by plant capacity (scale effects). For example, a hypothetical reverse osmosis system desalting feedwater at 4000 mg/ℓ to produce product water at 500 mg/ℓ would have cost characteristics as illustrated in Figure 2. These data are based on feedwater wells approximately one kilometer from the plant site and brine injection wells at the site. Interest, fuel, and electrical costs are those used by U. S. Department of the Interior (1972).

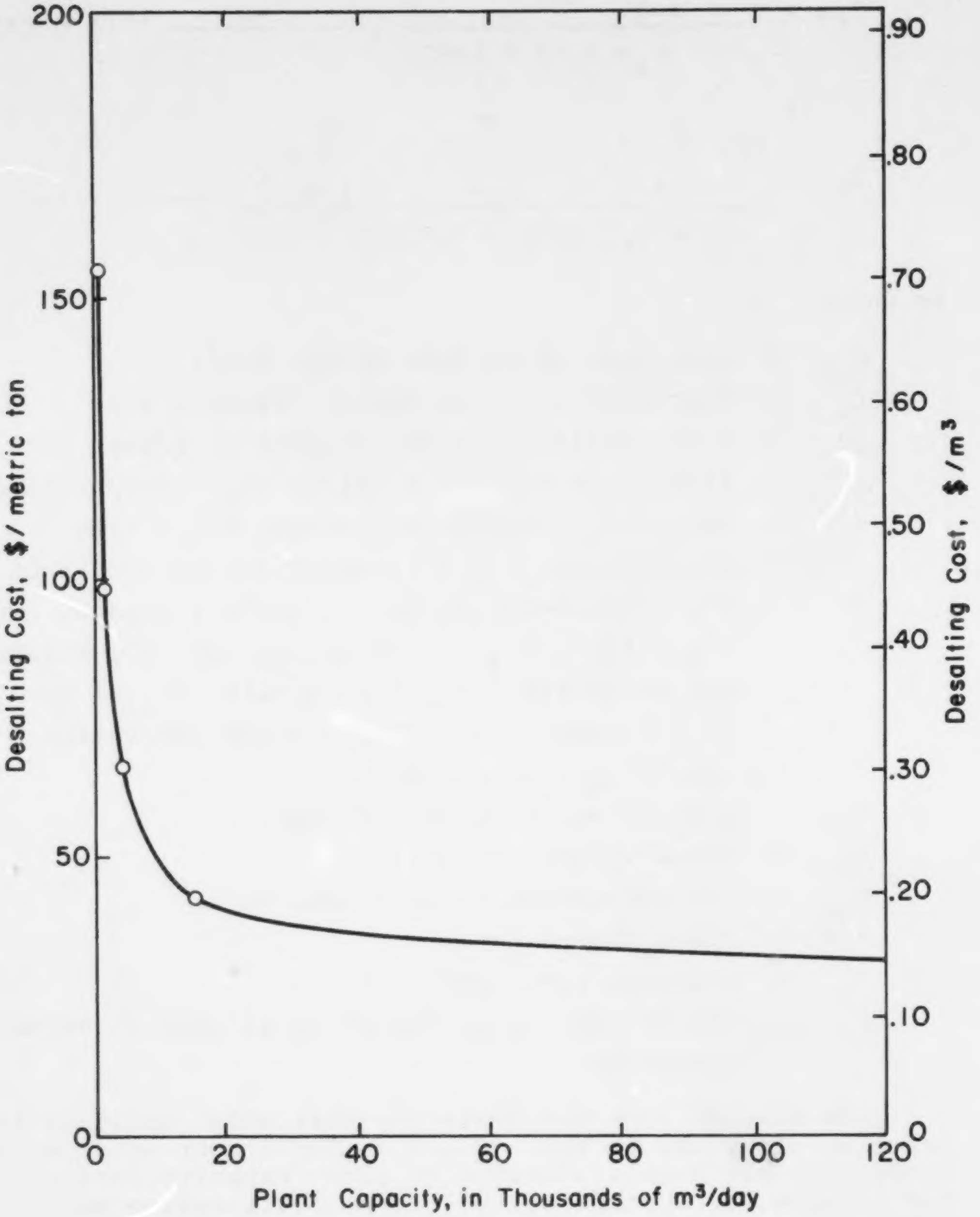


Figure 2. Desalting cost function for an RO system having feedwater at 4000 mg/ℓ and product water at 500 mg/ℓ.

PROCESS DESCRIPTION

Saline water conversion processess involve the use of a semi-permeable barrier, which exclude either water or salt flow. The barrier may be a membrane which excludes salt such as RO, one which excludes water such as ED, or one that exchanges salt for hydrogen and hydroxide ions which unite to produce water (IX). The barrier may also be a "phase boundary" which excludes the salts. For example, vaporization of water using MSF, VTE-MSF and VC-VTE-MSF leaves the salts in the remaining solution as does solidification of water using VF-VC processes (Probstein, 1973). The driving potential for each of these processes is either heat (distillation and freezing), pressure (RO), electrical (ED), or chemical (IX).

Each desalination process has specific advantages depending on such factors as feedwater chemistry and desired product water. A general review of these factors along with a description of the costing model for each technology will be given in this section. However, much of the intrinsic detail regarding operation characteristics or design requirements will be left to the interested reader to determine from available technical literature.

In most cases, a desalting system can be divided into feedwater, desalting, and brine disposal facilities. These three subsystems are integrated as shown schematically in Figure 3. Pre-and Post-treatment are considered part of the desalting plant facility.

Multi-stage Flash Distillation

In 1973 about 95% of the daily desalting capacity in the world was being accomplished by distillation (Probstein, 1973). Basically, distillation involves vaporizing a portion of the feedwater leaving the salts in a more concentrated environment of the remaining water (brine). Then the pure water vapor is condensed and removed as product water. A schematic view of this process is shown in Figure 4 for a staged system (U. S. Department of the Interior, 1972). In the MSF process, entering feedwater is heated under a pressure (>50 psi) to a temperature just under the boiling point and then injected into an expanded vessel having a reduced pressure. Part of the water then "flashes" or rapidly evaporates into steam at the water surfaces. Energy in the water vapor is then exchanged through a heat exchange with incoming feedwater to raise its temperature to the appropriate level. This process is repeated in succeeding stages at successively lower pressures. The evaporation within each stage is a function of pressure difference between stages, stage area, and flow rate. The MSF process is probably best applied to conditions where feedwaters are soft (carbonate

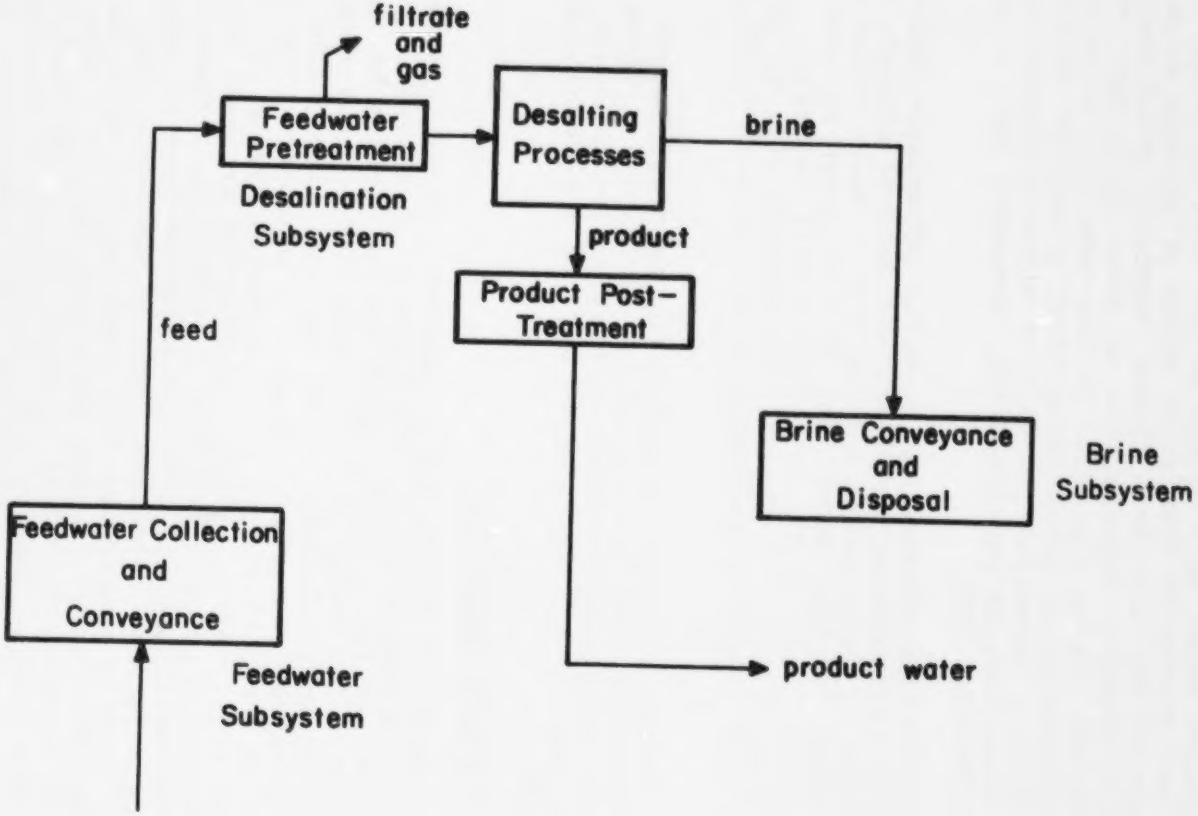

filtrate and gas
Feedwater Pretreatment
Desalination Subsystem
Desalting Processes
brine
product
Product Post-Treatment
feed
Brine Conveyance and Disposal
Brine Subsystem
Feedwater Collection and Conveyance
Feedwater Subsystem
product water

Figure 3. Schematic diagram of a typical desalination system.

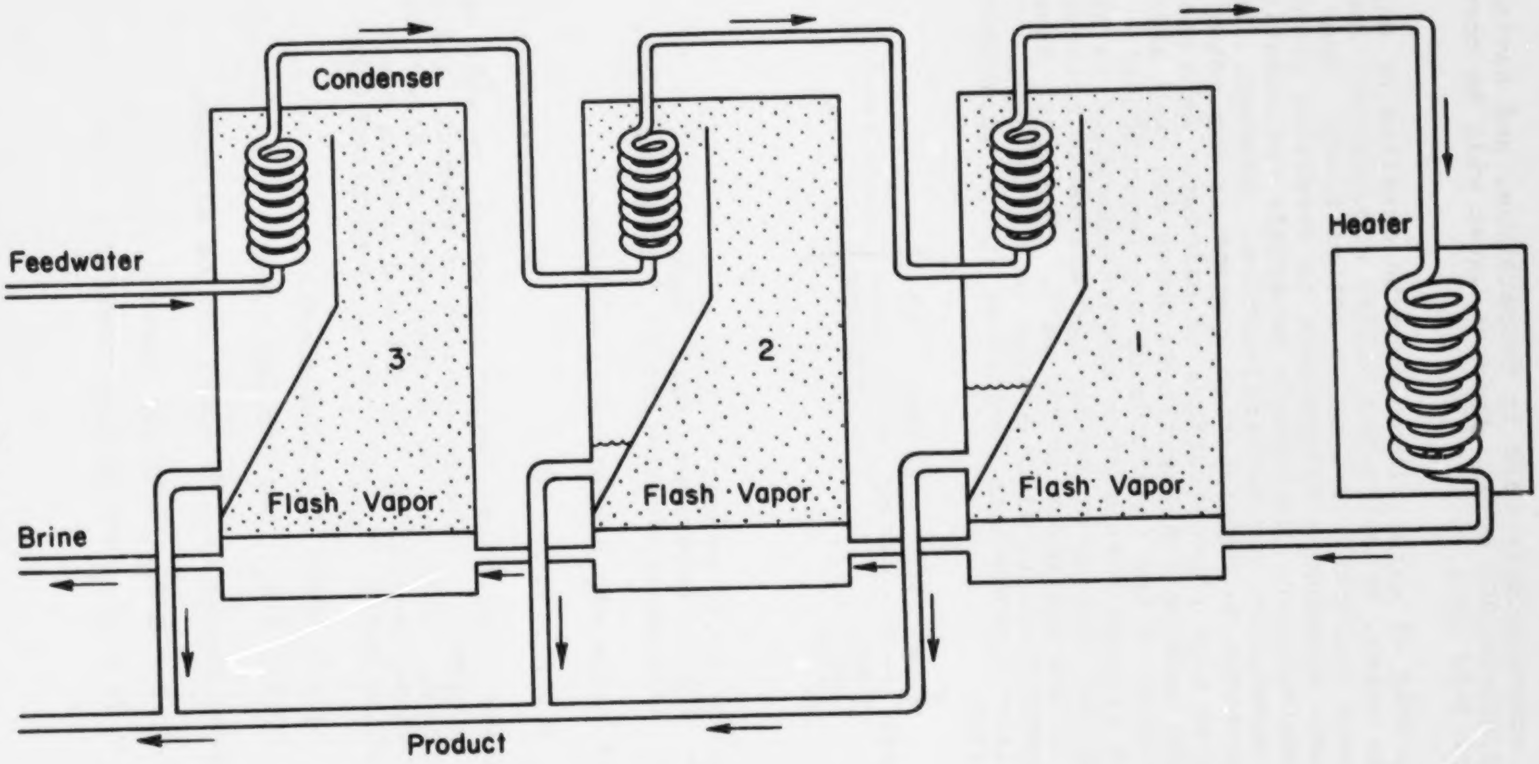

Condenser
Feedwater
3
2
1
Heater
Flash Vapor
Flash Vapor
Flash Vapor
Brine
Product

Figure 4. Diagram of typical Multi-Stage Flash Distillation systems.

hardness), comparatively cold in temperature, and having a TDS range of 10,000-50,000 mg/ℓ. Product water will be normally in the range of 5-50 mg/ℓ.

The variety of waters that might be desalted by any system includes sea water, brackish and saline groundwater, and brackish surface waters including irrigation return flows. Many of these waters contain substances deleterious to desalting plant operation. Dissolved gases and organic materials are usually controlled by deaeration and ultrafiltration. However, one of the principle problems in desalination systems is the potential for scaling due to high concentrations of calcium. As a general rule, waters having calcium concentrations above 600 mg/ℓ should be pretreated (such as the injection of a polyphosphate). In this study, it is assumed that sodium hexametaphosphate is utilized in all cases and that by so doing, the allowable calcium concentration in the feedwater in 900 mg/ℓ. The total dissolved solids concentration is also limited to 60,000 mg/ℓ. Thus, the ratio of brine to product is defined as (U. S. Department of the Interior, 1973):

$$BPR = \max \left[\frac{1-50/TDS_i}{\frac{900}{Ca_i} - 1} \text{ or } \frac{TDS_i - 50}{60{,}000 - TDS_i} \right] \quad \ldots\ldots\ldots\ldots (47)$$

in which,

BPR = brine to product ratio;
TDS_i = TDS in feedwater, mg/ℓ; and
Ca_i = calcium concentration in feedwater, mg/ℓ.

The volume of brine may therefore be written:

$$C_b = C_p \times BPR \quad \ldots\ldots\ldots\ldots (48)$$

Multi-stage flash distillation processes, as well as other distillation processes, require cooling of product (and possibly brine) discharges. This may be accomplished by direct exchange with cooling water or through the use of a cooling tower. The volume of cooling water (C_w, m^3/day) is determined from;

$$C_w = C_p\ (4.2 - BPR), \text{ (cooling tower)} \quad \ldots\ldots\ldots\ldots (49)$$

$$C_w = C_p\ (2.5 - BPR), \text{ (no cooling tower)} \quad \ldots\ldots\ldots\ldots (50)$$

The total system intake (C_i, m^3/day) can be written:

$$C_i = C_p\ (1.2 + BPR), \text{ (cooling towers)} \quad \ldots\ldots\ldots\ldots (51)$$

$$C_i = C_p + Cb + Cw, \text{ (no cooling towers)} \quad \ldots\ldots\ldots\ldots (52)$$

A mathematical simulation of the largely graphical procedures outlined by the U. S. Department of the Interior (1972) is summarized for a MSF desalting plant (including pretreatment and post-treatment) in Table 1. In estimating capital costs for construction, steam, site development, and land, the functions are based on product water capacity, C_p. The same is true for annual expenditures for labor and materials, chemicals, fuel, steam, and electricity. Interest during construction is determined by multiplying an estimate of construction time by one-half of the interest rate and applying this result to the sum of capital costs for construction, steam and site development. Start-up costs are assumed to be equal to one month of the sum of annual costs, whereas working capital is assumed to be twice the start-up costs. Costs for "owner's general expenses" are based on the level of investment in construction, steam facilities, and site development.

It should be noted that the equations in Table 1 are expressed in terms of English units in order to be congruent with the procedures in the source. Since this work is presented in metric units, the interested users of these models should multiply product water capacity expressed in m^3/day by 2.6417×10^{-4} to convert to million gallons per day (mgd). In addition, land prices in \$ million/ha need to be multiplied by 0.4047 to get \$ million/acre; fuel rates in \$/million joules (MJoules) need to be multiplied by 9.4787×10^{-4} to get \$/million BTU's (MBTU) and electrical rates in \$/MJoules need to be multiplied by 2.7778×10^{-4} to get \$/1000 kilowatt-hours (kwh).

Vertical Tube Evaporation - MSF

Another distillation process designed to maximize heat transfer efficiencies is the vertical tube evaporator (VTE) which has been by itself an alternative to MSF systems. However, hybrid VTE-MSF processes have been shown to improve thermodynamic efficiency, reduce brine pumping costs, and lower structural costs due to several common elements (U. S. Department of the Interior, 1972). Consequently, VTE by itself will not be included in this costing analysis although it is helpful to review its operation.

Like MSF systems, VTE processes make use of the principle that water will vaporize at a progressively lower temperature if associated also with progressively lower pressures. Unlike the MSF process, vapor produced in any stage is condensed in the succeeding stage to aid vapor formation therein (Figure 5). In the VTE-MSF process, feedwater is pumped alternately from MSF stages to VTE effects. The desalting system is best applied to the conditions noted earlier for the MSF process.

TABLE 1. SUMMARY OF COST FUNCTIONS FOR AN MSF DESALTING PLANT

Cost Description	Cost Function	Remarks
CAPITAL COSTS, $ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.)\times 1.48\times C_p^{.90456}$	$C_p\geq 50$ mgd
	$C_1=(ENR/952.)\times 1.90\times C_p^{.83817}$	$6\leq C_p\leq 50$
	$C_1=(ENR/952.)\times 2.30\times C_p^{.73444}$	$C_p\leq 6$
2 Steam Facilities, C_2	$C_2=(ENR/952.)\times 0.23\times C_p^{.8015}$	
3 Site Development Costs, C_3	$C_3=(ENR/952.)\times 0.10\times C_p^{.6365}$	
4 Interest During Constr., C_4	$C_4=15.5\times C_p^{.2119}\times I_r/24.\times(C_1+C_2+C_3)$	
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12{,}000$	
6 Owners' General Expense, C_6	$C_6=0.119(C_1+C_2+C_3)^{0.9}$	
7 Land Costs, C_7	$C_7=L_p(2.+.2168\times C_p+.377\times C_p^{.7756})$	
8 Working Capital, C_8	$C_8=2.0\times C_5$	
ANNUAL COSTS, $ THOUSANDS		
9 Labor Materials, C_9	$C_9=(BLS_1/3.76)\times 47.0\times C_p^{.6657}$	$C_p\geq 40$ mgd
	$C_9=(BLS_1/3.76)\times 60.0\times C_p^{.6014}$	$8\leq C_p\leq 40$
	$C_9=(BLS_1/3.76)\times 81.0\times C_p^{.4580}$	$3\leq C_p\leq 8$
	$C_9=(BLS_1/3.76)\times 95.0\times C_p^{.3042}$	$1\leq C_p\leq 3$
	$C_9=(BLS_1/3.76)\times 95.0\times C_p^{.1818}$	$C_p\leq 1$
10 Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)\times 7.3\times U_f\times C_p$	
11 Fuel, C_{11}	$C_{11}=F_r\times 445.3\times X_p^{.986}\times U_f$	$X_p=0.58\times C_p^{1.006}$
12 Steam, C_{12}	$C_{12}=(BLS_1/3.76)\times 64.0\times X_p^{.5476}\times U_f$	$X_p\geq 40$
	$C_{12}=(BLS_1/3.76)\times 70.0\times X_p^{0.5}\times U_f$	$6\leq X_p\leq 40$
	$C_{12}=(BLS_1/3.76)\times 78.0\times X_p^{.44898}\times U_f$	$1.5\leq X_p\leq 6$
	$C_{12}=(BLS_1/3.76)\times 80.0\times X_p^{.3571}\times U_f$	$X_p\leq 1.5$
13 Electricity, C_{13}	$C_{13}=E_c\times 3.65\times C_p^{.949}\times U_f$	
14 Replacement, C_{14}	$C_{14}=0.0$	

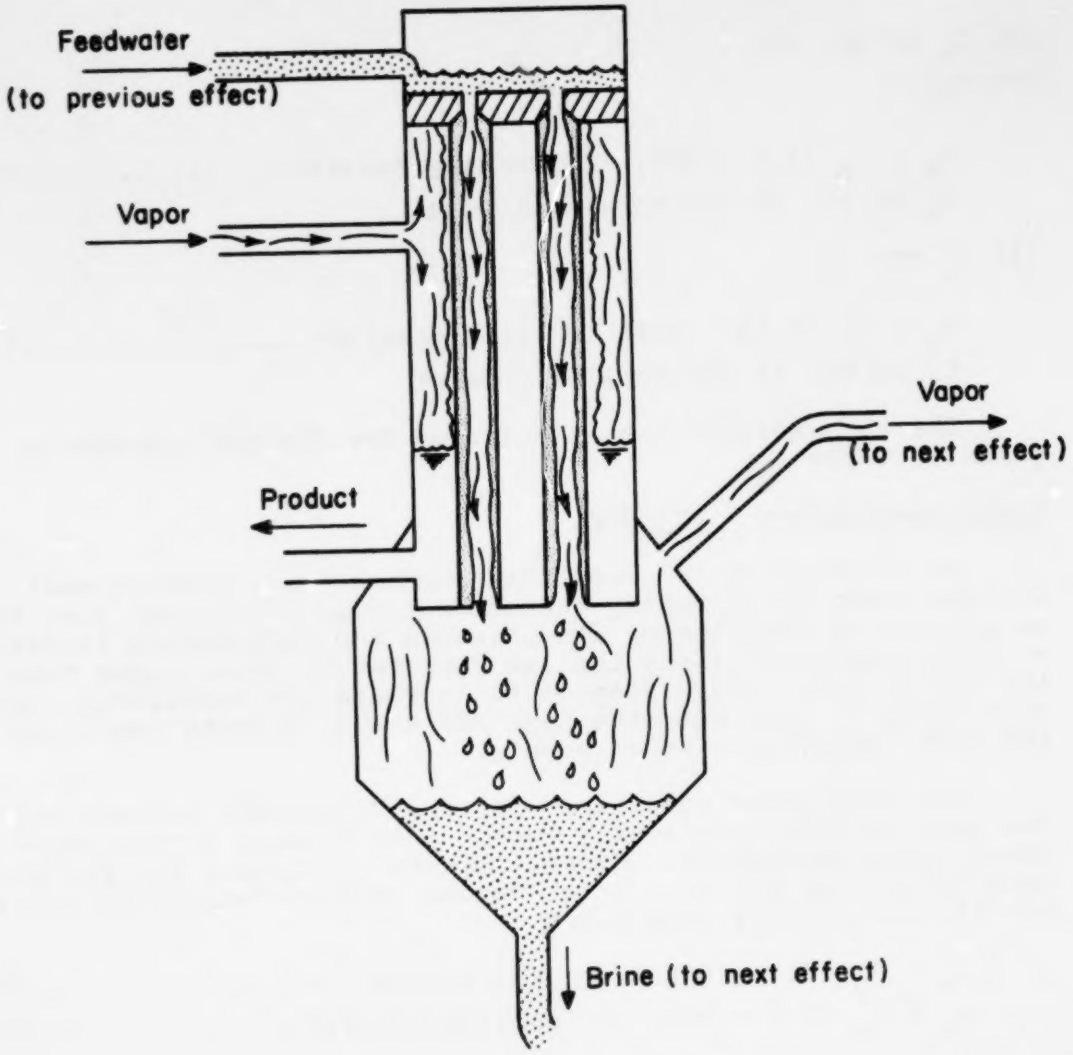

Feedwater
(to previous effect)
Vapor
Vapor
(to next effect)
Product
Brine (to next effect)

Figure 5. Illustration of the basic vertical tube evaporation effect.

Calculations of various flow rates and limitations include:

(1) BPR;

$$BPR = \max\left[\frac{1-(50/TDS_i)}{900/_{Ca_i} - 1} \text{ or } \frac{TDS_i - 50}{80{,}000 - TDS_i}\right] \qquad (53)$$

(2) C_b by Eq. 48;

(3) C_w;

$$C_w = C_p\ (3.2 - BPR)\ \text{(no cooling towers)} \qquad (54)$$

C_w by Eq. 50 (no cooling towers)

(4) C_i by:

$$C_i = C_p\ (1.15 + BPR)\ \text{(cooling towers)} \qquad (55)$$

C_i by Eq. 52 (no cooling towers)

The mathematical cost simulation for VTE-MSF process is given in Table 2.

Vapor Compression - VTE-MSF

An alternative to generating the necessary process heat through steam is to employ another enthalpy principle, i.e. that as a vapor is compressed, its pressure and temperature increase. A vapor compressor takes the low temperature water vapor from the VTE outlets, compresses it to increase the temperature and then feeds it back into the next MSF stage, thereby replacing the high temperature steam supply.

The applicable conditions for the VC-VTE-MSF process are the same as noted previously except this process favors relatively warm feedwaters. Hydraulic characteristics for BPR are determined from Eq. 53. Brine volume is then calculated using Eq. 48, and cooling waters by:

$$C_w = C_p\ (2.2 - BPR)\ \text{(cooling towers)} \qquad (56)$$

$$C_w = C_p\ (1.5 - BPR)\ \text{(no cooling towers)} \qquad (57)$$

Total intake rate is computed by:

$$C_i = C_p\ (1.1 + BPR) \qquad (58)$$

or for systems not using cooling towers, Eq. 52.

A summary of the cost simulation is given in Table 3.

TABLE 2. SUMMARY OF COST FUNCTIONS FOR VTE-MSF DESALTING PLANTS

Cost Description	Cost Functions	Remarks
CAPITAL COSTS, \$ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.\times1.5964\times C_p^{.8287}$	$C_p\geq25$ mgd
	$C_1=(ENR/952.)\times2.03965\times C_p^{.74537}$	$9\leq C_p\leq25$
	$C_1=(ENR/952.)\times2.57279\times C_p^{.64352}$	$C_p\leq9$
2 Steam Facilities, C_2	$C_2=(ENR/952.)\times0.23\times C_p^{.8015}$	
3 Site Development, C_3	$C_3=(ENR/952.)\times0.40\times C_p^{.6365}$	
4 Interest During Constr., C_4	$C_4=(C_1+C_2+C_3)\times I_r/24.\times14.0\times C_p^{.2389}$	
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12{,}000$	
6 Owners' General Expense, C_6	$C_6=0.119(C_1+C_2+C_3)^{0.90}$	
7 Land Costs, C_7	$C_7=L_p(2.+.2168\times C_p+.377\times C_p^{.7756})$	
8 Working Capital, C_8	$C_8=2.0\times C_5$	
ANNUAL COSTS, \$ THOUSANDS		
9 Labor-Materials, C_9	$C_9=(BLS_1/3.76)\times51.\times C_p^{.60979}$	$C_p\geq25$ mgd
	$C_9=(BLS_1/3.76)\times87.\times C_p^{.4476}$	$7\leq C_p\leq25$
	$C_9=(BLS_1/3.76)\times122.\times C_p^{.25874}$	$1.5\leq C_p\leq7$
	$C_9=(BLS_1/3.76)\times132.\times C_p^{.08392}$	$C_p\leq1.5$
10 Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)\times7.3\times U_f\times C_p$	
11 Fuel, C_{11}	$C_{11}=F_r\ 445.3\times X_p^{.986}\times U_f$	$X_p=0.58\times C_p^{1.006}$
12 Steam, C_{12}	$C_{12}=(BLS_1/3.76)\times64.\times X_p^{.5476}\times U_f$	$X_p\geq40$
	$C_{12}=(BLS_1/3.76)\times70.\times X_p^{0.5}\times U_f$	$6\leq X_p\leq40$
	$C_{12}=(BLS_1/3.76)\times78.\times X_p^{.44898}\times U_f$	$1.5\leq X_p\leq6$
	$C_{12}=(BLS_1/3.76)\times80.\times X_p^{.3571}\times U_f$	$X_p\leq1.5$
13 Electricity, C_{13}	$C_{13}=E_c\times3.65\times C_p^{.949}\times U_f$	
14 Replacement, C_{14}	$C_{14}=0.0$	

TABLE 3. SUMMARY OF COST FUNCTIONS FOR VC-VTE-MSF DESALTING PLANTS

Cost Description	Cost Functions	Remarks
CAPITAL COSTS, $ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.)\times 2.71\times C_p^{.7451}$	$C_p \geq 2$ mgd
	$C_1=(ENR/952.)\times 2.88\times C_p^{.64706}$	$1 \leq C_p \leq 2$
	$C_1=(ENR/952.)\times 2.88\times C_p^{.51961}$	$C_p \leq 1$
2 Steam Facilities, C_2	$C_2 = 0.0$	
3 Site Development, C_3	$C_3=(ENR/952.)\times 0.1\times C_p^{.6535}$	
4 Interest During Constr., C_4	$C_4=(C_1+C_2)\times I_r/24.\times(17.368+1.263\times C_p)$	
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12,000$	
6 Owners' General Expense, C_6	$C_6=.119(C_1+C_2+C_3)^{0.9}$	
7 Land Costs, C_7	$C_7=0.8\times L_p+0.2133\times L_p\times C_p$	
8 Working Capital, C_8	$C_8=2.0\times C_5$	
ANNUAL COSTS, $ THOUSANDS		
9 Labor-Materials, C_9	$C_9=(BLS_1/3.76)\times 46.\times C_p^{.83077}$	$C_p \geq 30$ mgd
	$C_9=(BLS_1/3.76)\times 80.\times C_p^{.6713}$	$6 \leq C_p \leq 30$
	$C_9=(BLS_1/3.76)\times 111.\times C_p^{.4797}$	$2 \leq C_p \leq 6$
	$C_9=(BLS_1/3.76)\times 122.\times C_p^{.36364}$	$1 \leq C_p \leq 2$
	$C_9=(BLS_1/3.76)\times 122.\times C_p^{.2797}$	$C_p \leq 1$
10 Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)\times U_f\times 7.3\times C_p$	
11 Fuel, C_{11}	$C_{11}=F_r\times 151.475\times C_p\times U_f$	
12 Steam, C_{12}	$C_{12}=0.0$	
13 Electricity, C_{13}	$C_{13}=0.0$	
14 Replacement, C_{14}	$C_{14}=0.0$	

Vacuum Freezing - Vapor Compression

A second basic desalting approach involving a phase change to separate salt and water is freezing. This process operates on the principle that saline water at the freezing point will simultaneously form pure water vapor and salt-free ice crystals. The ice crystals are then collected and melted by compressing the water vapor to a slightly higher pressure and temperature (U. S. Department of the Interior, 1972).

Although the prototype VF-VC systems are still small and some of the important technology remains to be completely developed, the process has a number of advantages over the previously discussed distillation processes. For instance, the VF-VC system operates at lower temperatures which minimizes corrosion and scaling and the heat transfer requirements are much lower. A schematic diagram of the VF-VC process is shown in Figure 6.

After a pretreatment step including deaeration and filtration, feedwater flow is divided and passed through two heat exchangers with brine and product water to cool the flow to almost the freezing point. The flow then enters the hydroconverter having a low atmospheric pressure where part of the water flashes into vapor and part is crystallized into ice. The brine and ice mixture is then pumped to the counterwash vessel. Since there is a differential in the density of the ice and brine, the ice aggragates at the top of the brine where it is washed by about 5% of the product water before being scraped from the top of the ice surface and returned to the hydroconverter vessel. The final process step in the hydroconverter involves compression of the water vapor until it condenses on the ice crystals. The ice also melts in this process and with the condensed water vapor is pumped from the system as the product water.

The VF-VC desalting process is most economically applied to cold feedwater sources having TDS concentrations of 5000-50,000 mg/ℓ. Product water is usually 300-500 mg/ℓ. The BPR is computed by:

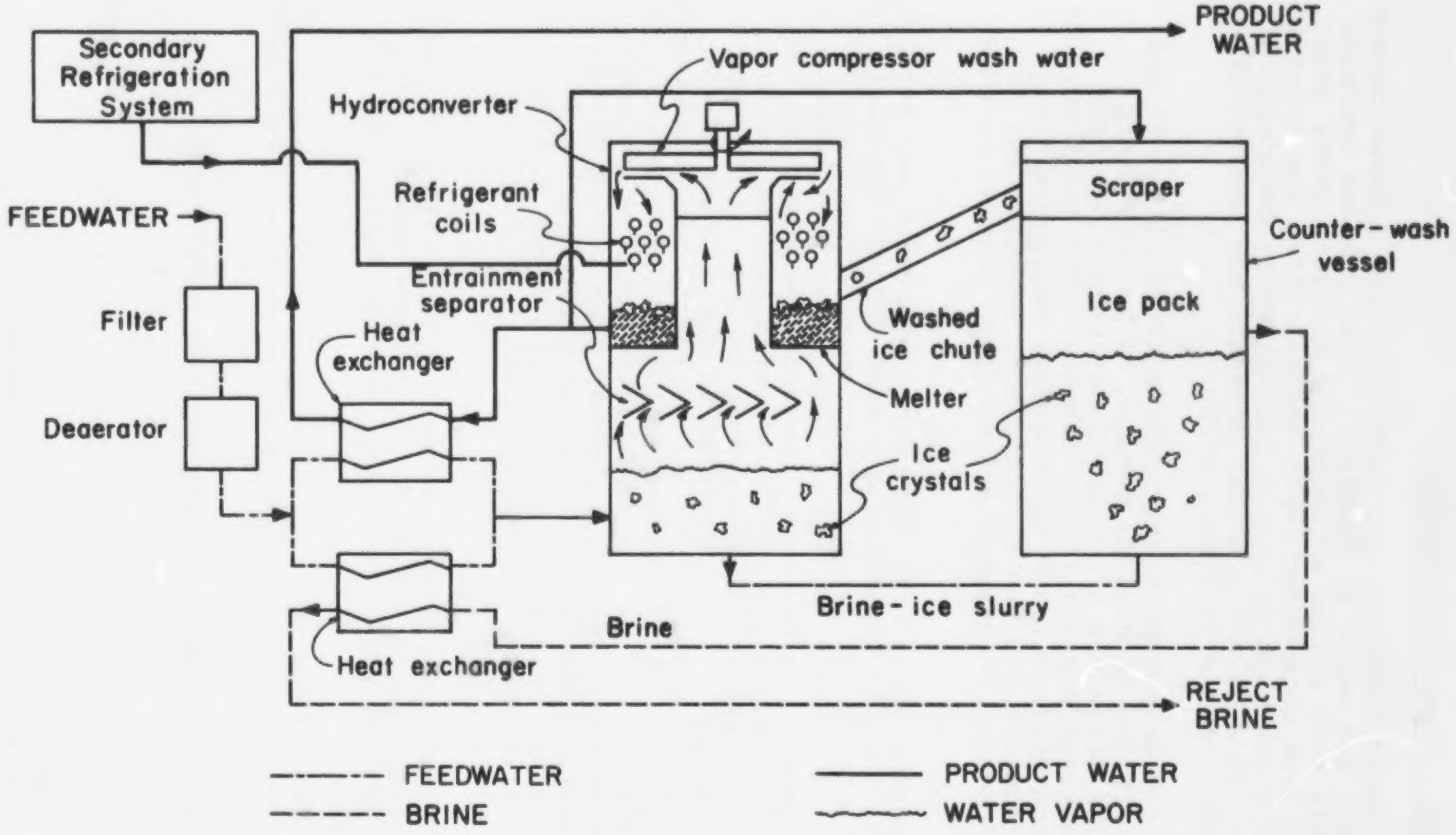
$$BPR = \frac{TDS_i - TDS_p}{60,000 - TDS_i} \quad \ldots\ldots (59)$$

and since no cooling water is required, the total intake rate is:

$$C_i = C_p + C_b \quad \ldots\ldots (60)$$

where C_b is determined from Eq. 48.

A summary of the costing model is given in Table 4.


Secondary Refrigeration System
FEEDWATER
Filter
Deaerator
PRODUCT WATER
Vapor compressor wash water
Hydroconverter
Refrigerant coils
Entrainment separator
Heat exchanger
Scraper
Counter-wash vessel
Ice pack
Washed ice chute
Melter
Ice crystals
Brine-ice slurry
Brine
Heat exchanger
REJECT BRINE
FEEDWATER
BRINE
PRODUCT WATER
WATER VAPOR

Vacuum Freezing – Vapor Compression Plant Schematic

Figure 6. Schematic diagram of VF-VC desalting process.

TABLE 4. SUMMARY OF COST FUNCTIONS FOR VF-VC DESALTING PLANTS

Cost Description	Cost Functions	Remarks
CAPITAL COSTS, $ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.)\times1.77\times C_p^{.8520}$	
2 Steam facilities, C_2	$C_2=0.0$	
3 Site Development, C_3	$C_3=(ENR/952.)\times0.1\times C_p^{.6535}$	
4 Interest During Constr., C_4	$C_4=(C_1+C_3)\times I_r/24.(17.368+1.263\times C_p)$	
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12,000$	
6 Owners' General Expense, C_6	$C_6=.119(C_1+C_3)^{0.9}$	
7 Land Costs, C_7	$C_7=L_p(0.8+0.32\times C_p)$	
8 Working Capital, C_8	$C_8=2.0\times C_5$	
ANNUAL COSTS, $ THOUSANDS		
9 Labor-Materials, C_9	$C_9=(BLS_1/3.76)\times123.\times C_p^{.64624}$	$C_p\geq7$ mgd
	$C_9=(BLS_1/3.76)\times132.\times C_p^{.5986}$	$C_p\leq7$
10 Chemicals, C_{10}	$C_{10}=0.0$	
11 Fuel, C_{11}	$C_{11}=0.0$	
12 Steam, C_{12}	$C_{12}=0.0$	
13 Electricity, C_{13}	$C_{13}=E_c(10.+.2375\times T+.0084375(TDS_i\times10^{-3})^2\times C_p^{.365}$	$C_p\geq.5$ mgd
	$C_{13}=C_{13}\times0.83/C_p^{.257}$	$C_p\leq0.5$
14 Replacement, C_{14}	$C_{14}=0.0$	

Electrodialysis

Electrodialysis removes salt from saline feedwaters by passing electrical current through positive and negative ion permeable membranes as illustrated in Figure 7. Unlike desalting processes involving high temperature or low temperature phase changes, the electrodialysis process uses energy at a rate proportional to the quantity of salts to be removed. Consequently, primary application of this technology is to soft, warm waters having 1000-5000 mg/ℓ of total dissolved solids. Product water is usually 300-500 mg/ℓ.

Feedwater entering a electrodialysis stack is divided into a brine flow and a product flow. Two electrodes on either side of the system created a positive potential to which the anions migrate and negative potential attracting cations. The brine and product flows are separated by an ion selective membrane allowing either cations or anions to pass but not both. The membranes are arranged as shown in Figure 7 to remove salts in the brine compartment. Passing individual flows through successive treatments allows production of product water at various levels of quality. Scaling and corrosion are major problems in electrodialysis systems and, therefore, require special attention, often by acidifying the brine side of the membranes.

The hydraulic limitations of the ED method follows about the same format as discussed earlier. The BPR, determined by,

$$\mathrm{BPR} = \frac{1-(\mathrm{TDS}_p/\mathrm{TDS}_i)}{(900/\mathrm{Ca}_i) - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(61)$$

must be greater than or equal to 0.15 due to electrical and chemical factors. Brine volume is determined by Eq. 48 and total intake rate by Eq. 60.

The costing model for ED plants is summarized in Table 5. As noted previously, the performance of the ED process as well as the costs are dependent on both the feed and product water quality. Estimates of construction, land and electrical costs are functions of the number of ED stages which involve first determining a "rating factor", RF, by:

$$\mathrm{RF} = \frac{0.575\{\mathrm{Na}_i + \mathrm{K}_i + \mathrm{Cl}_i\}}{\mathrm{TDS}_i} + 0.014375 \times (T-4.44) \quad \ldots\ldots(62)$$

where,

RF = plant rating factor;
Na_i = feedwater concentration of sodium, mg/ℓ;

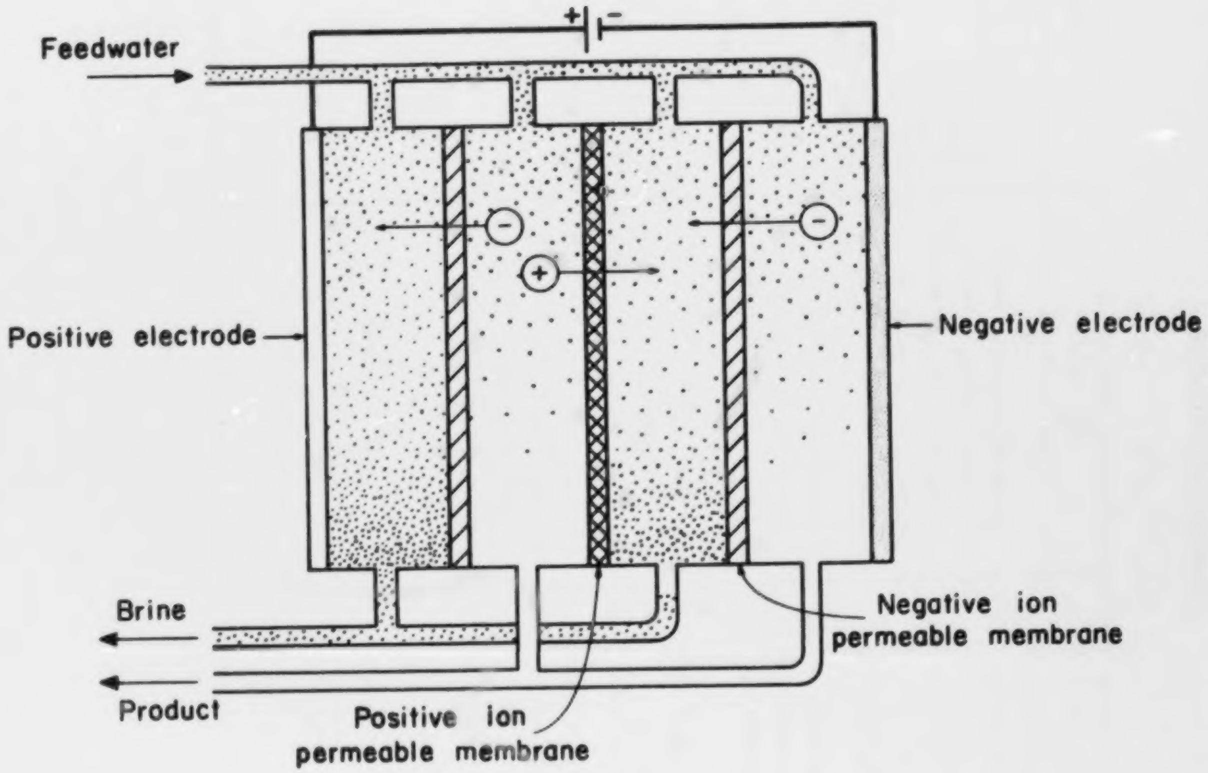

Feedwater
+
-
Positive electrode
Negative electrode
Brine
Product
Positive ion permeable membrane
Negative ion permeable membrane

Figure 7. Generalized view of a electrodialysis desalting process.

TABLE 5. SUMMARY OF COST FUNCTIONS FOR ED DESALTING PLANTS

Cost Description	Cost Functions	Remarks
CAPITAL COSTS, $ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.)\times.05194\times A_s^{.84962}$	$A_s \geq 80$
	$C_1=(ENR/952.)\times.08226\times A_s^{.75188}$	$25 \leq A_s \leq 80$
	$C_1=(ENR/952.)\times.1503\times A_s^{.56391}$	$4 \leq A_s \leq 25$
	$C_1=(ENR/952.)\times.203\times A_s^{.37594}$	$A_s \leq 4$
2 Steam Facilities, C_2	$C_2 = 0.0$	$A_s=\left\{\frac{\text{Log}(TDS_p)-\text{Log}(TDS_i)}{\log(FSR)}\right\}+1 \quad \left[\frac{C_p}{.252}+1\right]$
3 Site Development, C_3	$C_3=(ENR/952.)\times 0.1\times C_p^{.6535}$	
4 Interest During Constr., C_4	$C_4=(C_1+C_3)\times I_r/24.\times 8.\times C_p^{.3137}$	$FSR=0.53/RF^{.5418}$
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12{,}000$	$RF=\frac{.575(Na+K+Cl)}{TDS_i}+.014375\times(T-40.)$
6 Owners' General Expense, C_6	$C_6=.119(C_1+C_3)^{0.9}$	
7 Land Costs, C_7	$C_7=L_p(.0246\times A_s)$	
8 Working Capital, C_8	$C_8=2.0\times C_5$	
ANNUAL COSTS, $ THOUSANDS		
9 Labor-Materials, C_9	$C_9=(BLS_1/3.76)\times 25.5\times C_p^{.4726}+10.\times C_1$	$C_p \geq 1.5$ mgd
	$C_9=(BLS_1/3.76)\times 26.5\times C_p^{.4144}+10.\times C_1$	$Cp \leq 1.5$
10 Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)\times 18.25\times U_f\times C_p$	
11 Fuel, C_{11}	-0-	
12 Steam, C_{12}	-0-	
13 Electricity, C_{13}	$C_{13}=E_c\ \frac{20.47}{T\cdot 403}\times\ \frac{TDS_i}{1000}^{1.3042}\ C_p^{.365}\times U_f$	
14 Replacement, C_{14}	$C_{14}=1.6706\times A_s^{.9766}$	

K_i = feedwater concentration of potassium, mg/ℓ;
Cl_1 = feedwater concentration of chloride, mg/ℓ; and
T = feedwater temperature, degrees Celsius.

In general, ED stacks (or units as shown in Figure 7) are arranged in stages to achieve the desired product quality and in parallel rows to achieve the desired plant capacity. Thus, given the desired product quality and the rating factor, the number of stages (A_s) can be computed by first calculating the fraction of salts remaining after each stage, FSR:

$$FSR = 0.53/RF^{0.5418} \quad \text{.......} (63)$$

$$A_s = \left[\frac{\log(TDS_p) - \log(TDS_i)}{\log(FSR)} + 1\right]\left[\frac{C_p}{953.92} + 1\right] \quad \text{....} (64)$$

which assumes an individual stack capacity of 953.92 m³/day.

Reverse Osmosis

Another membrane process using hydraulic pressure rather than electrical potential to separate water and salts via a semi-permeable membrane is reverse osmosis. Much of the characteristics and problems noted earlier for ED processes apply to RO as well. For example, energy consumption is proportional to the quantity of salts to be removed. Consequently, the most economic application of RO plants should be to soft, warm feedwaters having 1000-10,000 mg/ℓ TDS and producing water of 100-500 mg/ℓ. The general RO plant flow network is illustrated in Figure 8.

Two vessels of water having different salt concentrations and separated by a semi-permeable membrane (permeable to water but exclusive of salts) will produce a flow of relatively pure water from the dilute solution to the more concentrated or until either they are both the same concentration or a buildup of pressure in the latter will stop the process. This phenomenon is called osmosis. It should also be noted that the more substantial the initial concentration differential, the greater the pressure (osmotic pressure) necessary to stop the flow. If a pressure greater than the osmotic pressure is applied to the solution of higher salinity, the flow of water can be reversed, thus the concept of reverse osmosis as illustrated in Figure 9.

The brine to product ratio for RO plants is computed from Eq. 61 but must exceed 0.11 due not only to scaling or fouling, but also because of existing membrane technology. It might be emphasized that this limitation will improve as new membranes are discovered and should therefore be evaluated periodically. Total intake rate and brine volume are determined from Eqs. 48 and 60.

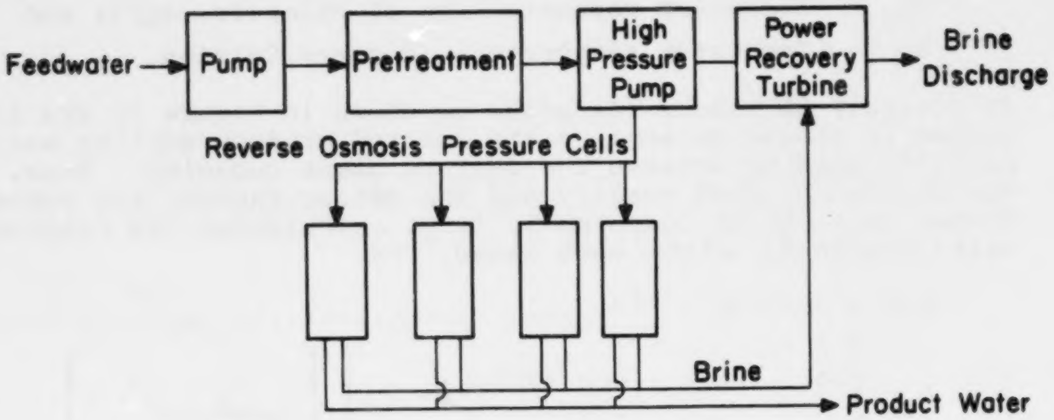

Feedwater
Pump
Pretreatment
High Pressure Pump
Power Recovery Turbine
Brine Discharge
Reverse Osmosis Pressure Cells
Brine
Product Water

Figure 8. Flow diagram of typical RO desalting system.

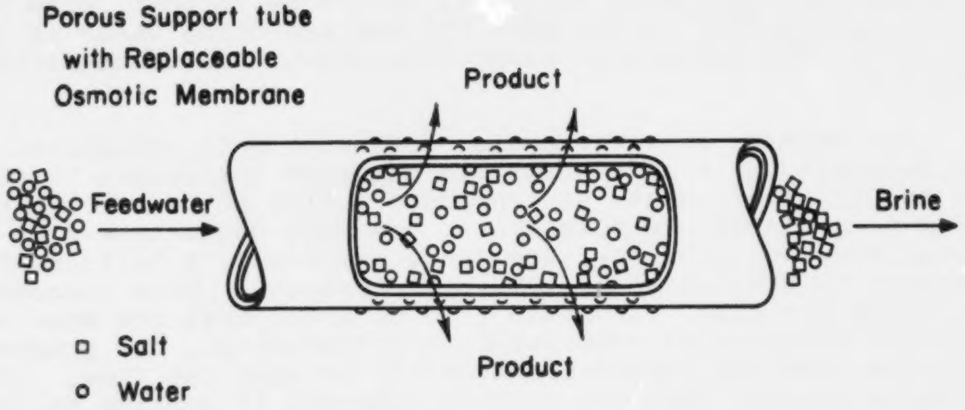

Porous Support tube with Replaceable Osmotic Membrane
Product
Feedwater
Brine
Product
□ Salt
○ Water

Figure 9. Illustration of basic RO process

The cost estimating relationships for the RO plants are given in Table 6. A temperature correction must be made to the product water capacity since the water permeation rate through the membranes decreases with decreasing feedwater temperatures. The corrected design capacity (X_t, m^3/day) is:

$$X_t = C_p + \frac{100 + 3.06\ (25 - T)}{100} \quad \text{.........................(65)}$$

Ion Exchange

The final desalting process included in this work is ion exchange, a process in which natural or synthetic resins having large quantities of exchangeable ions of a "beneficial" nature are used to exchange salinity ions in solution for ions on the resins. In a desalination context, the ions on the resins are H^+ and OH^- which when exchanged by the salts in the feedwater, unite to form water. Most economical applications of IX processes are for soft, warm feedwaters with salinities less than 2000 mg/ℓ and product water qualtities of 0-500 mg/ℓ. The IX process is probably most applicable where a relatively small amount of salts are to be removed and a high quality product is required. In addition, some advantage may be inherent in following one of the previously described methods with IX as a polishing step.

The IX desalting process generally utilizes a two stage system as shown in Figure 10. After pretreatment, feedwater is pumped into the "cation exchanger" consisting of an ion exchange resin using H^+ as the exchangeable ion. In this process, Na^+, Ca^{++}, Mg^{++}, and K^+ exchange with the H^+ on the resin producing a strongly acidic solution. Flow then proceeds to the "anion exchanger" where a resin utilizing OH^- as the exchangeable ion replaces NO_3^-, $SO_4^=$, and Cl^-. The addition of OH^- ions to the system neutralizes the solution leaving a high quality product water.

IX brine to product ratios are computed by:

$$BPR = (TDS_i - TDS_p)/1000 \quad \text{..............................(66)}$$

and total intake rate and brine volume by Eqs. 48 and 60, respectively.

The model of IX desalting costs is presented in Table 7 and it is seen that construction costs are a function of feedwater chemical characteristics. These relationships are developed by first expressing each anion present in terms of equivalent

TABLE 6. SUMMARY OF COST FUNCTIONS FOR RO DESALTING PLANTS

COST DESCRIPTION	COST FUNCTIONS	Remarks
Capital Costs, \$ Million		
1. Construction Costs, C_1	$C_1=(ENR/952.)\times.473\times X_t^{.91}$	$X_t\geq 8$
	$C_1=(ENR/952.)\times.522\times X_t^{.85774}$	$4\leq X_t\leq 8$
	$C_1=(ENR/952.)\times.575\times X_t^{.79184}$	$X_t\leq 4$
2. Steam Facilities, C_2	$C_2=0.0$	$X_t=C_p+\frac{(100.+1.7(77.-T))}{100.}$
3. Site Development, C_3	$C_3=(ENR/952.)\times 0.1\times C_p^{.6535}$	
4. Interest During Construction, C_4	$C_4=(C_1+C_3)\times I_r/24.\times 8.\times C_p^{.3137}$	
5. Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{13})/12,000.$	
6. Owners' General Expense, C_6	$C_6=.119(C_1+C_3)^{.9}$	
7. Land Costs, C_7	$C_7=L_p(.5+.298\times C_p)$	
8. Working Capital, C_8	$C_8=2.0\times C_5$	
Annual Costs, \$ Thousands		
9. Labor & Materials, C_9	$C_9=(BLS_1/3.76)\times 25.5\times C_p^{.4726}$	$C_p\geq 1.5$
	$C_9=(BLS_1/3.76)\times 26.5\times C_p^{.4144}$	$C_p\leq 1.5$
10. Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)\times 18.25\times U_f\times C_p$	
11. Fuel Costs, C_{11}	$C_{11}=0.0$	
12. Steam, C_{12}	$C_{12}=0.0$	
13 Electricity, C_{13}	$C_{13}=E_c\times 3.65\times C_p\times U_f$	
14. Replacement, C_{14}	$C_{14}=36.5\times U_f\times C_p$	

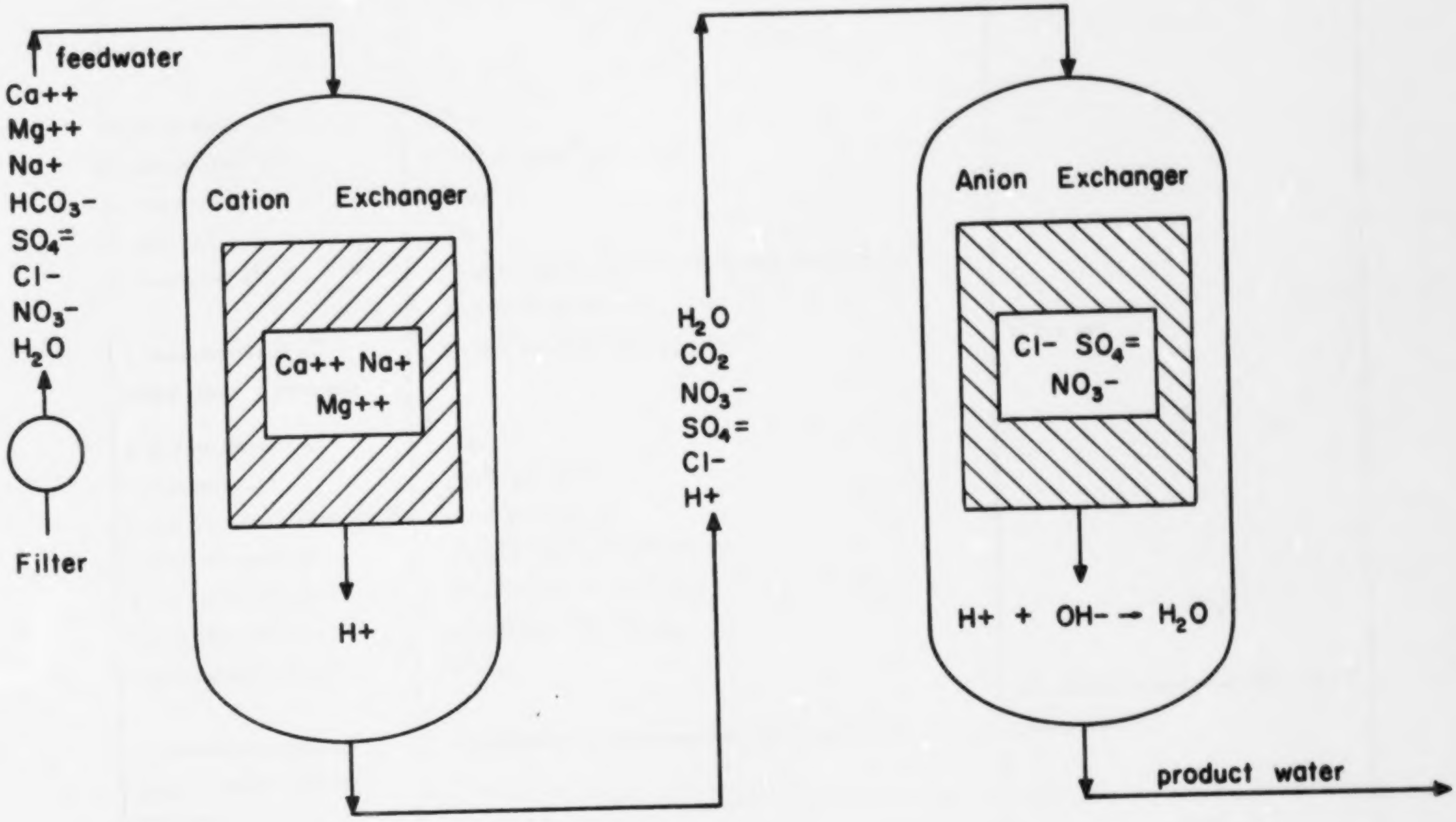

feedwater
Ca^{++}
Mg^{++}
Na^{+}
HCO_3^-
$SO_4^{=}$
Cl^-
NO_3^-
H_2O
Filter
Cation Exchanger
Ca^{++} Na^{+}
Mg^{++}
H^+
H_2O
CO_2
NO_3^-
$SO_4^{=}$
Cl^-
H^+
Anion Exchanger
Cl^- $SO_4^{=}$
NO_3^-
$H^+ + OH^- \rightarrow H_2O$
product water

Figure 10. Ion exchange desalting process.

TABLE 7. SUMMARY OF COST FUNCTIONS FOR IX DESALTING PLANTS

Cost Description	Cost Functions	Remarks
CAPITAL COSTS, $ MILLION		
1 Construction Costs, C_1	$C_1=(ENR/952.)*(.8572-.4595*BAR)((TDS_i-TDS_p)C_p*10^{-3})^{.6979}$	$BAR=.82*HCO_3/Y_p$ $Y_p=.82*HCO_3+1.41*Cl+.81*NO_3+1.04*SO_4$
2 Steam Facilities, C_2	$C_2=0.0$	
3 Site Development, C_3	$C_3=(ENR/952.)*0.1*C_p^{.6535}$	
4 Interest During Constr., C_4	$C_4=(C_1+C_3)*I_r/24.(8.*C_p^{.3137})$	
5 Start-up Costs, C_5	$C_5=(C_9+C_{10}+C_{11}+C_{12}+C_{13})/12,000$	
6 Owner's General Expense, C_6	$C_6=.119(C_1+C_3)^{0.9}$	
7 Land Costs, C_7	$C_7=L_p(.8+0.32*C_p)$	
8 Working Capital, C_8	$C_8=2.*C_5$	
ANNUAL COSTS, $ THOUSANDS		
9 Labor-Materials, C_9	$C_9=(BLS_1/3.76)*25.5*C_p^{.4726}+10.*C_1$	$C_p \geq 1.5$ mgd
	$C_9=(BLS_1/3.76)*26.5*C_p^{.4144}+10.*C_1$	$C_p \leq 1.5$
10 Chemicals, C_{10}	$C_{10}=(BLS_2/104.4)*3.65(.023-.01696\ BAR)(TDS_i-TDS_p)*C_p$	
11 Fuel, C_{11}	$C_{11}=0.0$	
12 Steam, C_{12}	$C_{12}=0.0$	
13 Electricity, C_{13}	$C_{13}=E_c(1.0514*C_p^{.8533})*.365$	
14 Replacement, C_{14}	$C_{14}=30.*C_1$	

$CaCO_3$ concentrations:

$$Y_p = 0.82xHCO_3 + 1.41xCl + 0.81xNO_3 + 1.04xSO_4 \quad \ldots\ldots\ldots\ldots (67)$$

where Y_p is the total anion concentration expressed as $CaCO_3$. Then, the bicarbonate ion ratio, BAR, is determined:

$$BAR = (0.82 \times HCO3_i) \cdot Y_p \quad \ldots\ldots\ldots\ldots (68)$$

which is subsequently utilized in costing procedures as illustrated.

FEEDWATER AND BRINE DISPOSAL

In a typical desalting complex, a significant fraction of the annual expenditures is associated with facilities to collect and convey feedwater, and to convey and dispose of brines. Feedwater facilities in this study are limited in several respects. First, the water supply to be desalted is assumed to be either groundwater which can be collected with well fields or surface flow capable of simple diversion. (No costs have been attributed to surface diversions.) After collection, feedwater is conveyed by concrete pipeline which may require pumping stations to satisfy both the transmission and desalting plant head requirements. Pipeline capacity for cost estimating purposes is considered to be the capacity of the desalting complex, C_p, or if cooling water must also be carried, $C_w + C_p$. The length of the pipeline is assumed to be the weighted average connector serving the individual wells or surface diversions. The relatively high cost incurred by these assumptions should insure a conservative estimate of desalting costs.

Brine disposal is assumed to be accomplished by either injection wells or evaporation ponds. Brine is also conveyed by concrete pipeline.

It should be noted that cost estimates for desalting plant facilities discussed in the previous section do not utilize cooling towers. Consequently, cooling towers have been added as an option in the development of feedwater and brine disposal systems.

Feedwater Wells

The costs of groundwater wells for supplying feedwater to desalting complexes are primarily a function of number and capacity of wells and the well depth. For this study, individual well capacities are assumed to be approximately 18,900 m^3/day so that the number of wells needed would be:

$$ANFW = (C_p / 18{,}900) + 1 \quad \ldots\ldots\ldots\ldots (69)$$

where,

ANFW = number of feedwater wells.

Cooling water is assumed to be supplied to conveyance pipelines from surface sources.

Pumping systems are often needed at the well site to bring the groundwater to the surface. Costs for this equipment are assumed to be a function of well capacity ~~and are included~~ in well costs.

A summary of the feedwater costing model along with the other facilities for feedwater supply, cooling, and brine disposal is given in Table 8 for capital costs and Table 9 for the annual operation and maintenance costs.

Pipeline & Pipeline Pumping Plants

As noted earlier, all conveyance to and from the desalting plant is facilitated by concrete piplines whose costs are a function of capacity, C_p' and length, D (D = D_{if} for feedwater and D = D_{ib} for brine disposal). Included in the cost of the well field are the surface facilities to pump water from the wells into the pipeline. Consequently, energy to supply conveyance headloss is assumed to be met by a pumping plant somewhere along the pipelines. The model, however, does not include elevational headloss where the desalting plant and well field are at different levels. The pipeline pumping plant costs are functions of both capacity and pumping head as illustrated in Tables 8 and 9.

Evaporation Ponds

An alternative for disposing of desalting plant wastes is to convey the brine to a large open area where they can be ponded in shallow areas and evaporated. The costs of evaporation ponds are primarily functions of pond area. The area of the pond is determined by:

$$E_{pa} = \frac{365\ C_b}{E} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (70)$$

in which,

E_{pa} = evaporation pond area, ha; and
E = mean annual evaporation rate, m.

As can be seen, areas in which evaporation rates are low or land prices are high due to urbanization will probably find evaporation pond costs too high.

TABLE 8. SUMMARY OF CAPITAL CONSTRUCTION COSTS FUNCTIONS FOR FEEDWATER, COOLING AND BRINE DISPOSAL FACILITIES, IN $ MILLION

Cost Description	Cost Functions	Remarks
1 Construction Costs, C_1		
a Feedwater Wells	$C_{1a}=(ENR/952.)*ANFW((.3+.6875*D_f)*0.41(C_p/ANFW)^{.4225})$	
	$+(PMI/1.41)ANFW((1.6+.48*C_p)+(1.4+1.316*C_p))/1000$	$ANFW=(C_p/5.)+1$
b Pipelines	$C_{1b}=(CPI/1.19)3.4*C'_p{}^{.7164}*D*10^{-6}$	$C'_p \ge 7$ mgd, $C'_p=C_p+C_w$
	$C_{1b}=(CPI/1.17)5.*C'_p{}^{.5442}*D*10^{-6}$	$2\le C'_p \le 7$
	$C_{1b}=(CPI/1.17)5.8*C'_p{}^{.3503}*D*10^{-6}$	$.6\le C'_p \le 2$
	$C_{1b}=(CPI/1.17)5.5*C'_p{}^{.2007}*D*10^{-6}$	$C_p\le .6$
c Pipeline Pumping Plants	$C_{1c}=(PPI/1.26)H_k*5.8*C'_p{}^{.9703}*10^{-3}$	$H_k=.07163*H^{.6548}$, $H\ge 150$ feet
		$H_k=.1978*H^{.4667}$, $80\le H\le 150$
		$H_k=.6637*H^{.178}$, $H\le 80$
		$H=(D_{if}/5280.)147.*C'_p{}^{-.773}$, $C'_p \ge 15$ mgd
		$H=(D_{if}/5280.)71.*C'_p{}^{-.515}$. $4\le C'_p \le 15$
		$H=(D_{if}/5280.)58.\ C'_p{}^{-.339}$, $C'_p \le 4$
d Evaporation Ponds	$C_{1d}=(EI/1.27)6.333*E_{pa}$	$E_{pa}=1.12*C_b/E$
e Injection Wells	$C_{1e}=(ENR/952.)A_k*.153*C_b^{.9867}$	$C_b\ge 2.5$ mgd
	$C_{1e}=(ENR/952.)A_k*.192*C_b^{.748}$	$.8\le C_b\le 2.5$
	$C_{1e}=(ENR/952.)A_k*.188*C_b^{.4258}$	$.3\le C_b\le .8$
	$C_{1e}=(ENR/952.)A_k*.128*C_b^{.1011}$	$C_b\le .3$
		$A_k=.32+.3692*D_b*10^{-3}$, $D_b\ge 6000$ feet
		$A_k=.6+.3167*D_b*10^{-3}$, $2000\le D_b\le 6000$
		$A_k=.8+.2367*D_b*10^{-3}$, $D_b\le 2000$
f Injection Well Surface Facilities	$C_{1f}=(PMI/1.41)*.29*C_b$	$C_b\ge 5$ mgd
	$C_{1f}=(PMI/1.41)*(.2+.2525*C_b)$	$2\le C_b\le 5$
	$C_{1f}=(PMI/1.41)*(.3+.23*C_b)$	$C_b\le 2$
g Cooling towers	$C_{1g}=(ENR/952.)*(.019*C_w^{.854})$	
2 Steam Facilities; 3. Site Development; 5. Start-up Costs; 8. Working Capital.	$C_2=C_3=C_5=C_8=0.0$	
4 Interest During Constr., C_4	$C_4=C_1*I_r/2.0$	
6 Owners' General Expense, C_6	$C_6=.119*C_1^{.9}$	
7 Land Costs, C_7	$C_7=L_p(A_1+A_2+A_3+A_4+A_5+A_6)$	$A_1=ANFW/2.0$; $A_2=1.+.0417*C_w$
8 Working Capital, C_8	$C_8=(C_9+C_{13})/6000$	$A_3=D_f*(1.5+.0063C'_p)/5280$
		$A_4=C_b/2.$; $A_5=E_{pa}$
		$A_6=D_{ib}*(1.5+.0063*C_b)/5280$

TABLE 9. SUMMARY OF ANNUAL O & M COST FUNCTIONS FOR FEEDWATER. COOLING, AND BRINE DISPOSAL FACILITIES IN $ THOUSANDS

Cost Description	Cost Functions	Remarks
9 Labor & Materials, C_9		
a Feedwater Well Systems	$C_{9a}=(BLS_1/3.76)\times 21.xC_p^{.869}$	$C_p \geq 1$ mgd
	$C_{9a}=(BLS_1/3.76)\times 21.xC_p^{.6818}$	$.25 \leq C_p \leq 1$
	$C_{9a}(BLS_1/3.76)x16.xC_p^{.4982}$	$C_p \leq .25$
b Pipelines	$C_{9b}=0.0$	
c Pipeline Pumping Plants	$C_{9c}=(BLS_1/3.76)x1.68xC_p'^{.7546}$	$C_p' \geq 1.5$ mgd
	$C_{9c}=(BLS_1/3.76)x1.76xC_p^{.6557}$	$.25 \leq C_p' \leq 1.5$
	$C_{9c}=(BLS_1/3.76)x1.44xC_p'^{.5255}$	$C_p' \leq .25$
d Evaporation Ponds	$C_{9d}=.0075xC_{1d}$	$C_p' = C_p+C_w$
e Injection Wells	$C_{9e}=(BLS_1/3.76)x44xC_b'^{.8728}$	$C_b \geq 2$ mgd
	$C_{9e}=(BLS_1/3.76)x50.xC_b^{.7644}$	$.5 \leq C_b \leq 2$
	$C_{9e}=(BLS_1/3.76)x44.xC_b^{.4479}$	$C_b \leq .5$
f Injection Well Surface Facilities	$C_{9f}=0.0$	
g Cooling Towers	$C_{9g}=10.C_{1g}$	
10 Chemicals, C_{10}, 11. Fuel, C_{11}; 12. Steam, C_{12}; 14. Replacement, C_{14}	$C_{10}=C_{11}=C_{12}=C_{14}=0.0$	
13 Electricity, C_{13}	$C_{13}=E_c x365.x10^{-6}x.004x(H_f+H_b)$	$H_f=(D_{if}xY/5280.+200.)C_p'x\,10^3$ $Y=147.C_p'^{-.773}\ C_p \geq 15$ mgd $Y=147.xC_p'^{-.515}\ 4 \leq C_p \leq 15$ $Y=58.xC_p'^{-339}\ C_p \leq 4$ $H_b=(D_{ib}\ Y'/5280.+200.)C_b x10^3$ $Y'=147.xC_b^{-.773}\ C_b \geq .15$ mgd $Y'=71.xC_b^{-.515}\ 4 \leq C_b \leq 15$ $Y'=58.xC_b^{-.339}\ C_b \leq 4$

Injection Wells and Surface Facilities

In many areas it may be possible to drill wells into deep aquifers of saline water (which do not interact with the surface hydrology or groundwater supply system). Such wells can be utilized as a disposal alternative for desalting brines by injecting them through the well into the deep aquifer. Costs for injection well systems, including surface facilities, depend on well depth and capacity as shown in Tables 8 and 9.

DESALTING COST ANALYSIS

The application of desalting technology to regional water quality management tends to be a very site specific problem. As a result, generalization of cost analyses is difficult. However, it might be useful to point out the model's sensitivity to various input parameters relative to an arbitrary "base" so the relative importance of the variables can be viewed. Table 10 summarizes the base values of input parameters utilized in this section.

Desalting System Capacity

In a previous paragraph it was mentioned that desalting costs expressed in terms of dollars per ton of salt removed or dollars per cubic meter of product water exhibit substantial economics of scale. For the base condition, the scale effects for each process are illustrated in Figure 11 (ion exchange has not been included because of the high TDS in the feedwater). In nearly every process, the costs at 950 m^3/day are 2-4 times the costs at 121,000 m^3/day. The VTE-MSF, VC-VTE-MSF, and RO process costs at the lower valve are 3.5 to 3.6 times the upper capacity indicating much larger importance of scale than is associated with MSF (2.80), VF-VC (2.78), or ED (2.13). Of these specific processes, electrodialysis is more affected by input parameters and therefore should be evaluated more closely in the definite plan investigation.

The scale factor in desalination will generally preclude small installations for salinity control since other measures of reducing salinity will be cheaper. However, as the level of implementation increases and the desalting costs decrease, this technology may become highly competitive with the various other alternatives. Consequently, a major parameter in a salintiy control analysis that can be expected to affect the potential use of desalination is this level of implementation.

Feedwater Salinity

Because of referencing desalting costs to salinity control, the feedwater salinity is an important parameter in the evaluation of the alternative processes. The distillation and

TABLE 10. STANDARDIZED DESALTING MODEL INPUT PARAMETERS FOR VARIABLE PARAMETER SENSITIVITY ANALYSES

C_p = 1.5×10^4 m^3/day (4 mgd)
TDS_i = 5000 mg/ℓ
TDS_p = 500 mg/ℓ
ENR = 1354
BLS_1 = 4.93
BLS_2 = 181
CPI = 1.88
PPI = 1.98
PMI = 2.13
EI = 1.92
D_b = 1000 m (3280 feet)
D_f = 100 m (328 feet)
D_{ib} = 100 m (328 feet)
D_{if} = 1000 m (3280 feet)
E = 1.07 m (3.5 feet)
E_c = \$$7.2\times10^4$/M Joules (\$20/1000 kwh)
FCR = 0.0856
F_r = \$$1.2\times10^3$/M Joules (\$1.14/MBTU)
I_r = 7%
L_p = \$4,942/ha (\$2000/acre)
T = 15.6 C (60 F)
U_f = 0.90
Na = 1260 mg/ℓ
Mg = 123 mg/ℓ
Ca = 393 mg/ℓ
K = 8 mg/ℓ
HCO_3 = 106 mg/ℓ
Cl = 2035 mg/ℓ
SO_4 = 1075 mg/ℓ
NO_3 = 0 mg/ℓ

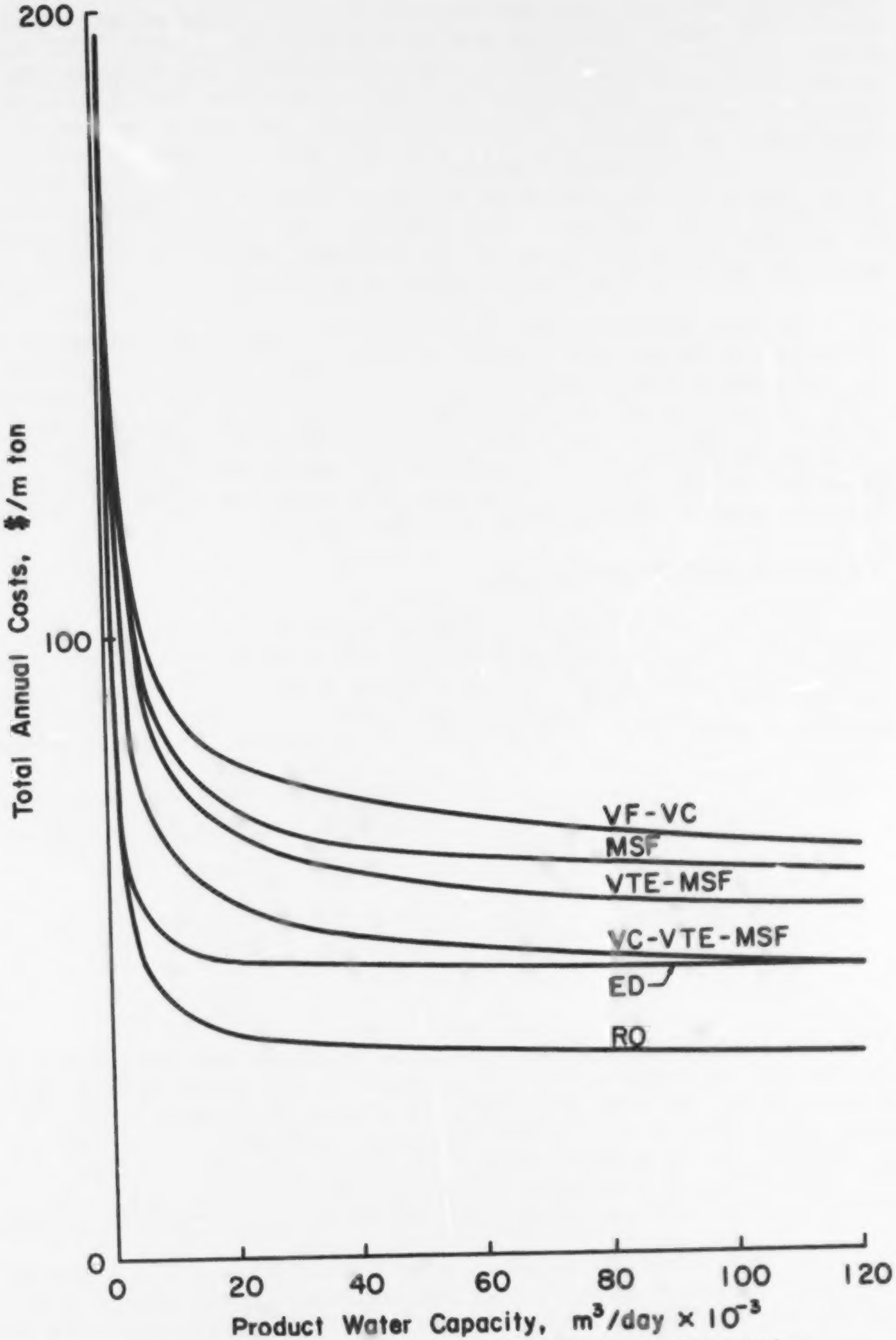


Figure 11. Relationship of plant capacity and desalting costs for various systems.

freezing methods are not substantially limited by input salinity since the same measures are necessary to desalt 1000 mg/ℓ feedwater as for 10,000 mg/ℓ. Consequently, the higher the feedwater salinity the lower the unit cost for these processes. Reverse osmosis and electrodialysis, on the other hand, use membranes to effect a salt removal and therefore are directly affected by feedwater salinity. Calculations were made at the base input condition with various levels of input salinity to evaluate this factor. The results are plotted in Figure 12. It should be noted that it is assumed that individual ionic species do not create limiting conditions.

In all the methods simulated in the costing model except electrodialysis, the costs at 2000 mg/ℓ would be double those at 5000 mg/ℓ and five times those at 13,000 mg/ℓ. However, the rate of change of the salinity versus cost ratio diminished toward increasing salinity values. The electrodialysis process is significantly less affected by feedwater salinity mainly because of its module construction and the direct relationship between power consumption and salinity.

Operation and Maintenance

Unlike the generally predictable construction cost items, operation and maintenance costs are subject to year-to-year inflational pressures which cannot always be effectively predicted. At the base condition, the O & M costs were typically somewhat more than 50% of the total annual cost, as shown below.

Process	Percent of Total Annual Cost Attributable to O & M
MSF	60%
VTE-MSF	57
VC-VTE-MSF	46
ED	58
RO	56
VF-VC	56

It is interesting to evaluate the effects of interest rate on the relative importance of operation and maintenance costs. For example, if the interest rate was increased from 7% at the base to 10.5%, O & M costs decline from more than 50% to 39%, or about 11% in most cases. In terms of total costs, this 50% increase in interest rates increased unit costs by a low of 21.5% for MSF processes to 28.6% for VC-VTE-MSF systems.

In each case studied, the effects of scale or feedwater salinity did not affect the percentages given above with the exception of electrodialysis. In the ED analysis, the effects of plant capacity produced O & M percentages ranging from 58% to 66% as the capacity increased from the base to 121,000 m^3/day.

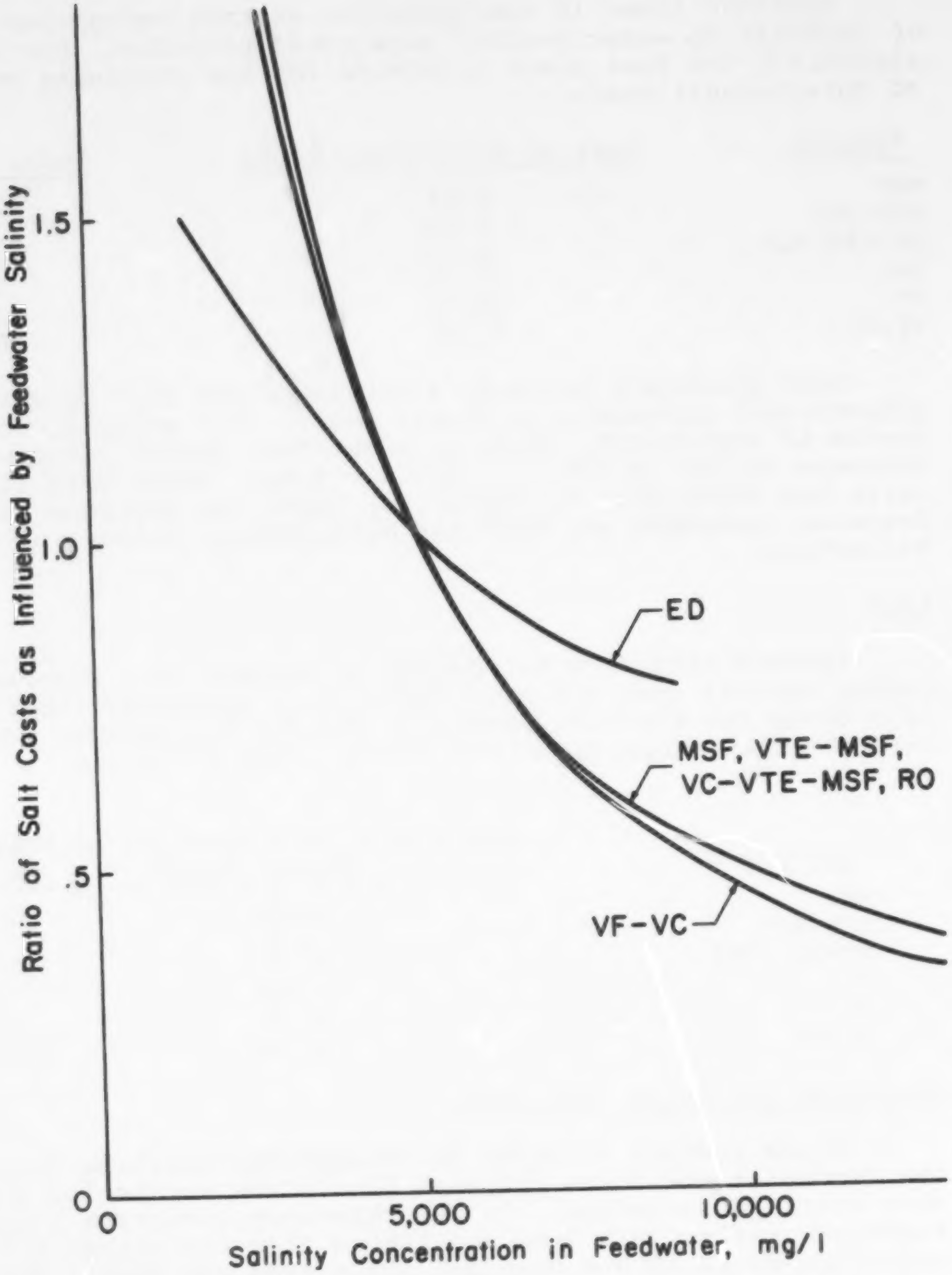


Figure 12. Effects of feedwater salinity concentration on desalting costs.

Specific items in the operating expense categories are also of interest to water quality management planning. For instance, electrical and fuel costs accounted for the following percentages of total annual costs:

Process	Cost of Electrical Power	Costs of Fuel
MSF	9.8%	32.0%
VTE-MSF	5.4%	33.4%
VC-CTE-MSF	2.1%	24.0%
ED	27.5%	0 %
RO	20.4%	0 %
VF-VC	32.8%	0 %

Rate increases for both electricity and fuel produce proportional increases in annual costs. For example, in a MSF system if electricity rates increase 50%, annual costs will increase (0.50) (0.098) = 0.049, or 4.9%. These data illustrate the importance of electrical costs for membrane and freezing processes and fuel for distillation methods of desalination.

Land

Because land area for desalting systems is a non-depreciating capital cost and can therefore be amortized indefinitely, land costs for the base condition ($5,000/ha) account for only 1-8% of total annual desalting costs. Specifically:

Process	Land Costs as a Fraction of Total Annual Costs
MSF	3.8%
VTE-MSF	3.9%
VC-VTE-MSF	4.8%
ED	5.8%
RO	7.7%
VF-VC	0.7%

Feedwater and Brine Facilities

Of the factors involved in evaluating desalting feasibility, the feedwater and brine disposal facilities may be the most site-specific variables. In the reference situation using feedwater wells 100 m deep and 1000 m from the plant, and brine injection wells 1000 m deep and 100 m from the plant, the costs were as follows:

Process	Percentage of Total Annual Costs Attributable to Feedwater and Brine Disposal Facilities
MSF	21%
VTE-MSF	21%
VC-VTE-MSF	25%
ED	30%
RO	40%
VF-VC	12%

Differences in the values reflect different volumes of feedwater, brine, and overall costs.

The choice of a feedwater or brine disposal system is very important in evaluating these alternatives. Feedwater wells, for example, can account for about 15% of the costs of feedwater and brine disposal systems. Likewise, the choice of brine injection wells over evaporation ponds can be a significant decision. In the base example, evaporation ponds would cost about 70% more than injection wells for each alternative except VF-VC in which case they would cost approximately 7% less. Thus, the selection and location of these facilities should be considered and optimized for each potential location.

Technological Advances

The desalting submodel does not include compensation for technological improvement in equipment or processes that are almost certain to appear. The potential users of this work should therefore be cognizant that substantial errors can be introduced if updating with current information is not part of using this desalting submodel.

SECTION 6

SIMULATION OF AGRICULTURAL SALINITY CONTROL COSTS

INTRODUCTION

Improved management and structural rehabilitation are often regarded as the most feasible treatments of an irrigation system to improve the quality of return flow. Indirect approaches such as limiting irrigation diversions, effluent standards, land use regulations, and economic incentives may also be considered although they appear more difficult to implement. Whenever salt pickup is the objective of salinity control, however, the specific control measures should impact segments of the irrigation system which contribute to the magnitude of local groundwater flow. This may be accomplished by reducing seepage from various elements of the conveyance system and minimizing deep percolation from over-irrigation. Return flow quality may also be improved by relief and interceptor drainage to collect subsurface flows before a chemical equilibrium is reached with the ambient soil or aquifer materials.

The agricultural salinity control cost model is composed of cost-effectiveness functions for each alternative input to the groundwater region where salt pickup is assumed to occur. These are based on soil and aquifer chemical behavior as interpreted by prerequisite analyses. Thus, the relationship between groundwater flow and salt loading can be developed in such a manner that control costs can be related to reductions in salt loading.

The model developed in this section is divided into two categories: (1) analysis of the conveyance system; and (2) analysis of the farm level irrigation system.

WATER CONVEYANCE SYSTEM ANALYSIS

The contribution to local or regional salinity problems from irrigation water conveyance networks may be the result of a number of factors. First, unlined channels allow seepage into underlying soils and aquifers where naturally occurring salts might be dissolved and transported into receiving waters. Second, the structural and managerial condition of a system may support large areas of open water surfaces or phreatophytes which concentrate salinity in return flows. Finally, the operation of the system may preclude efficient water utilization by the individual irrigator, especially if deliveries are not made in accordance with crop demands. The costs to remedy these problems are generally limited to those associated with lining and rehabilitation of the waterway. However, so far as improved management may be required, some costs associated with educational programs and legal/administrative adjustments may be incurred.

To test the feasibility of conveyance system improvements relative to other salinity control measures, the mathematical model developed in the following paragraphs will include two principal channel improvement alternatives; concrete linings and piping. It is assumed that these alternatives along with supportive structures represent the generally applicable technology in terms of both utilization and cost. It is further assumed that converting an unlined conveyance channel to a buried pipe will be limited to networks having less than 0.25 m^3/sec capacity since pipes available in the larger sizes require special fabrication and therefore cost much more than the concrete lining alternative.

Concrete Lined Systems

Seepage from canal, lateral, and ditch conveyance networks may be reduced or eliminated by lining the perimeter with an impervious material such as concrete, plastic, asphalt, or compacted earth to note several of the more common methods. Concrete is probably the most commonly employed lining material because of the combined advantages of cost, ease of construction, availability, reduced maintenance, and low permeability. The costs of concrete linings (either slip-form or gunite) vary with local economic and topographic conditions, channel geometry and size, and requirements for miscellaneous water management, safety, and environmental structures. For specific locations it is important to prepare cost estimates on a case-by-case basis, although for planning purposes it is useful to have generalized expressions.

A review of concrete lining cost by Walker (1976) indicated that such costs could be reasonably well estimated as a function of wetted perimeter and updated to present and future conditions with an appropriate cost index. A simpler methodology based on design discharge can also be utilized, and will be the variable in this model.

Data presented by U. S. Department of Interior (1963, 1975) and Evans, et al.(1976) were evaluated by the following general relationship:

$$U_c = K_1 \cdot Q^{K_2} \quad \text{.......................................} (71)$$

where,

U_c = unit lining cost \$/m;
Q = design discharge, m^3/sec; and
K_1, K_2 = regression coefficients.

After transforming the data with the Bureau of Reclamation canal and earthwork cost index to a base time of January 1976, the K_1 and K_2 values were 29.70 and 0.56, respectively. It should be noted, however, that even in the same locale these unit costs varied substantially. Equation 71, therefore, is intended only as a general estimating formula. The unit costs in Eq. 71 include only the earthwork, relocation, and lining costs and do not include costs for fencing, diversion structures, safety structures, etc. The latter costs are also highly variable depending upon the many site-specific conditions. An examination of such costs as given by the U. S. Department of Interior (1975) showed a range of \$12/m to \$50/m with a average of \$22/m. Unless otherwise specified, the average figure will be used in this analysis. Thus, the per meter construction costs, C_c, may be written in 1976 dollars as:

$$C_c = 29.7Q^{0.56} + 22 \quad \text{.................................} (72)$$

In addition to the construction costs, one must consider service facilities, engineering, investigations, and other administrative expenses. The Bureau of Reclamation has used factors of approximately 35% for these costs, so Eq. 72 can be written as a total capital cost as:

$$C_c = 40.1Q^{0.56} + 29.70 \quad \text{...............................} (73)$$

In order to calculate the total costs for a given length of canal, ditch, or lateral, Eq. 73 must be integrated over the applicable limits. Because water is continually being withdrawn from a conveyance channel, both wetted perimeter and discharge decline along the length of the channel. Assuming a linear decline for the decrease, the wetted perimeter at a specific location can be determined by:

$$WP = WP_m(1-bL/L_t) \quad \text{..................................} (74)$$

in which,

WP_m = the wetted perimeter at the channel inlet, m;

L = length from inlet to specified point, m;
L_t = total length of channel, m; and
b = empirical constant representing the fraction of maximum wetted perimeter remaining at end of the channel.

Similarly for the design discharge:

$$Q = Q_m \, (1-bL/L_t) \qquad (75)$$

where,

Q_m = inlet channel capacity in m^3/sec.

Combining Eqs. 73 - 75, yields,

$$C_c = 40.1 \cdot Q_m^{0.56} (1-bL/L_t)^{0.56} + 29.70 \qquad (76)$$

Then, the total capital costs for lining L meters of channel $\overline{C}_c$, are determined by integrating Eq. 76 over length as it varies from 0 to L meters:

$$\overline{C}_c = 40.1 \cdot Q_m^{0.56} \frac{L_t}{1.56 \cdot b} \left[1-(1-bL/L_t)^{1.56}\right] + 29.7 \cdot L \qquad (77)$$

Equation 77 assumes that the lining proceeds in the downstream direction, however, the choice of either upstream or downstream lining direction depends on their relative cost-effectiveness. To determine this choice, it is first necessary to define the salinity control effectiveness resulting from a particular lining project. The difference between the equilibrium salinity in return flow and the salinity in the seepage water represents the volume of salt pickup by the seepage losses and thereby, the salinity control expected from the linings. The volume of salt loading affected by a reduction in seepage can be written:

$$\Delta S_1 = \Delta S_c \cdot \left[V_s \, \frac{Q_g - Q_p}{Q_g} \right] \cdot 10^{-6} \qquad (78)$$

where,

ΔS_1 = reduction in salt loading due to the linings, metric tons/m annually;
ΔS_c = difference between the equilibrium salinity concentration in the return flows and the salt concentrations in the seepage waters, mg/ℓ;
Q_g = total groundwater additions, $m^3/year$;

Q_p = phreatophyte use of groundwater, m^3/year; and
V_s = total volume of seepage, m^3/m^2/year, as determined by:

$$V_s = N_d \cdot \Delta SR \cdot WP' \quad \ldots\ldots (79)$$

in which,

N_d, = number of days per year seepage occurs;
WP = wetted perimeter of original channel, m; and
ΔSR = change in seepage rate affected by lining, m^3/m^2/day.

It might be pointed out that the ΔSR value might also be written as a length distributed parameter in this model if measurements or other data are available.

The question of lining direction can be addressed by approximating the marginal costs at both ends and then comparing the results. First, let K_1, K_2, and K_3 be defined respectively as:

$$K_1 = 40.1 \cdot Q_m^{0.56} \quad \ldots\ldots (80)$$

$$K_2 = \Delta S_c \cdot N_d \cdot \Delta SR \left[\frac{Q_g - Q_p}{Q_g}\right] \cdot WP'_m \cdot 10^{-6} \quad \ldots\ldots (81)$$

$$K_3 = 29.7 \quad \ldots\ldots (82)$$

Then the marginal cost estimate would be found by substituting Eq. 74 and 79 into Eq. 78 and then dividing Eq. 76 by the results. After simplifying, the marginal cost is written as:

$$MC = \frac{K_1(1-bL/L_t)^{0.56} + K_3}{K_2(1-bL/L_t)} \quad \ldots\ldots (83)$$

in which,

MC = marginal lining cost estimates, \$/ton.

At the inlet when L = 0 and the marginal cost estimate, MC_i, is,

$$MC_i = \frac{K_1 + K_3}{K_2} \quad \ldots\ldots (84)$$

whereas at the end where $L = L_t$, the marginal cost estimate, MC_e, is:

$$MC_e = \frac{K_1(1 - b)^{0.56} + K_3}{K_2(1-b)} \quad \ldots\ldots (85)$$

Subtracting Eq. 85 from 84 and simplifying gives:

$$MC_i - MC_e = \frac{K_1}{K_2}\left[1 - \frac{1}{(1-b)^{0.44}}\right] + \frac{K_3}{K_2}\left[1 - \frac{1}{(1-b)}\right] \quad \ldots\ldots (86)$$

Because b is a value between 0 and 1, Eq. 86 is always negative indicating that the optimal direction is downstream.

The cost-effectiveness functions for concrete channel linings in the salinity control sense, as determined by first integrating the expression, result from substitution of Eqs. 74 and 79 into 78. The results for the downstream lining can be written:

$$S_1 = K_2 L(1-bL/2L_t) \quad \ldots\ldots (87)$$

in which,

S_1 = salt load reduction after lining L meters of conveyance channel, m tons/year.

If L is solved for in Eq. 87 and substituted into Eq. 77, the cost of linings are expressed as a function of the expected reduction in salt loading:

$$\bar{C}_c = K_i \left[1 - (1-b)\cdot f(S_1)/L_t)^{1.56}\right] + 29.7 \cdot f(S_1) \quad \ldots\ldots (88)$$

where,

$$K_i = \frac{K_1 \; L_t}{1.56 \cdot b} \quad \ldots\ldots (89)$$

and,

$$f(S_1) = \frac{L_t}{b} - \left[\left(\frac{L_t}{b}\right)^2 - \frac{2L_t}{K_2 b} \cdot S_1\right]^{0.5} \quad \ldots\ldots (90)$$

Buried Plastic Pipeline

For conveyances with discharge capacity up to 0.25 m³/sec, an alternative to concrete lining is buried plastic pipeline. Pipelines offer a number of advantages over concrete, particularly with respect to ease of installation, higher seepage loss reductions, and less interference with surface traffic. In irrigation system applications, low head plastic pipe is most often used and in fact, as will be noted later, is only economically competitive with concrete with the low head specifications.

The discharge capacity of buried plastic pipelines is a function of the pipe diameter and the head loss per unit length. According to the commonly employed Hazen-Williams equation,

$$J = 4.35 \times 10^{17} \left(\frac{Q}{150}\right)^{1.852} D^{-4.87} \quad \ldots\ldots\ldots\ldots\ldots\ldots (91)$$

where,

J = the head loss in m per 100 m;
Q = pipe flow in m^3/sec; and
D = pipe diameter in mm.

The constant, 150, represents a generally accepted friction coefficient for continuous lengths of plastic pipe.

The usual design decision with respect to pipeline head losses in irrigation systems would be to approximate the natural land slope in the direction of the pipeline. Equation 91 can then be written in terms of diameter,

$$D = 622.77\, Q^{0.38} J^{-0.21} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (92)$$

Cost data for low head PVC pipelines in western Colorado indicate installation costs averaging \$3.78 per meter. When these expenses were added to January 1976 pipe costs, it was possible to develop a polynomial expression as a function of discharge,

$$C_{pl} = 5.77 - 9.8466Q^{.38}J^{-0.21} + 31.05Q^{0.76}J^{-0.42} \quad \ldots\ldots\ldots (93)$$

in which,

C_{pl} = pipelinecosts (pipe & installation) \$/m;
Q = pipeline capacity, m /sec; and
J = pipeline slope, m/100m.

If thecosts of engineering, negotiations, etc., are included as was part of the canal and lateral concrete lining costs (35%), C_{pl} would be,

$$C_{pl} = 8.88 - 15.15Q^{0.38}J^{-0.21} + 47.76Q^{0.76}J^{-0.42} \quad \ldots\ldots\ldots (94)$$

It is interesting to compare the respective feasibility of plastic pipeline as opposed to concrete lined channel as determined from earlier equations (for channels this small, K_3 is assumed to be zero in Eqs. 76 and 77). A plot of the comparison for three possible pipeline slopes covering a fairly representative range is given in Figure 13. The concrete lined sections exhibit substantially better economics of scale than do the pipelines and are generally less costly at the small sizes. Pipelines appear in the plot to be most attractive for the large slopes.

The salinity control cost-effectiveness of buried plastic pipeline can be determined by combining a modified form of Eq. 87 with the integrated form of Eq. 94. An important assumption might be made in this regard. Conveyance systems with this comparatively small capacity would not be expected to have a decreasing capacity along their length as was the case for the major conveyance described earlier. Generally, these kinds of systems serve as laterals, head-ditches or tailwater ditches where capacity is not designed to diminish since the full water supply must be available to the irrigators at the lower ends of these systems. In addition, these capacities may very well fall within the limits of the smallest available size of pipe or concrete ditch. Under these conditions, the reduction in salt loading derived from Eq. 87 would simplify to:

$$S_1 = K_2 \cdot L \quad \ldots\ldots (95)$$

The integrated form of Eq. 94 is:

$$\bar{C}_{pl} = C_{pl} \int_0^L dL = C_{pl} \cdot L \quad \ldots\ldots (96)$$

Thus solving for L in Eq. 95 and substituting into Eq. 96 gives:

$$\bar{C}_{pl} = \frac{8.88S_1}{K_2} - \frac{15.15Q^{0.38}J^{-0.21}S_1}{K_2} + \frac{47.76Q^{0.76}J^{-0.21}S_1}{K_2} \quad \ldots\ldots (97)$$

ON-FARM SYSTEM ANALYSIS

The water applied as irrigation to croplands takes several routes back to the groundwater and stream systems from which it was diverted or into the atmosphere where it originated. Every segment of this complex system, except the atmospheric transition (evaporation and transpiration) is characterized by a chemical constitutent derived from the earth and rock materials contacted by the water.

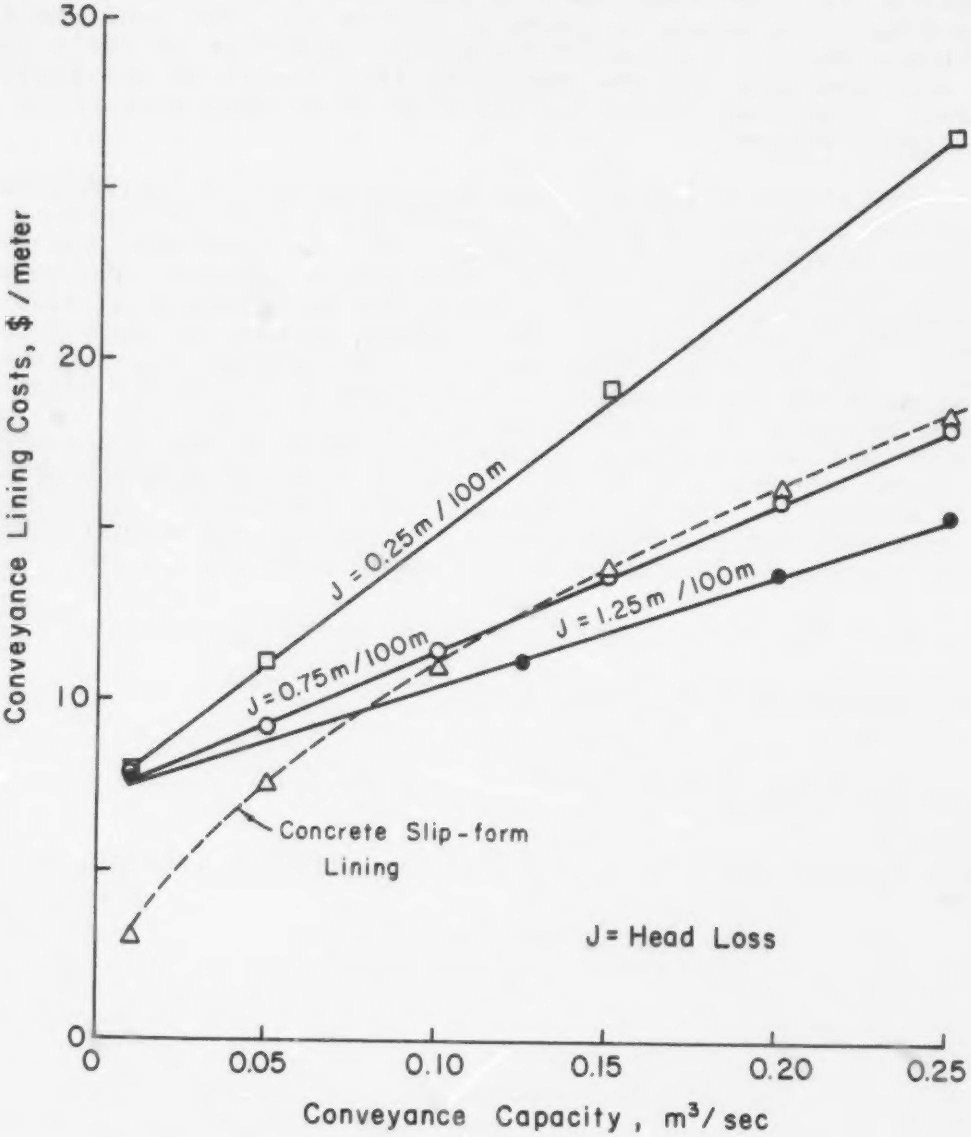


Figure 13. Comparison of concrete and buried plastic pipeline lining costs.

Salinity is primarily associated with two segments of the field hydrology, deep percolation and on-farm conveyance seepage. The measure of effectiveness in managing deep percolation and seepage is generally irrigation efficiency. However, this term is too broad for use in on-farm analyses. Rather, a more specific term, application efficiency (AE), provides a more resolved indication of control on percolation and seepage losses. Application efficiency is the percentage of irrigation waters actually applied to the soil reservoir that is stored and then utilized from the root zone. Another on-farm waste, field tailwater, is not included in this definition. Precipitation entering the soil profile should be included. Thus,

$$AE = \frac{\Delta S_m + E_T'}{I + P} \cdot 100 = \frac{E_T}{F_d - T_w + P} \cdot 100 \quad \cdots\cdots\cdots\cdots (98)$$

where,

ΔS_m = change in soil moisture storage before and after an irrigation, cm;
I = infiltrated irrigation depth, cm;
P = precipitation during the irrigation, cm;
E_T' = evapotranspiration during the irrigation, cm;
E_T = evapotranspiration between irrigations, cm;
F_d = field deliveries, cm; and
T_w = field tailwater, cm.

Leaching is not included in this definition because of the largely external view of salinity being applied.

The salinity associated with farm related irrigation return flows will be minimized when application efficiency is maximized. Alternatives for accomplishing an increase in AE can be divided into two basic types. The first type is the array of non-structural management and operational practices which provide closer coordination between water applications and available soil moisture storage capacity. Many of these practices (Irrigation timing, amounts to apply, pesticide applications, etc.) are incorporated in one technology called irrigation scheduling. Unfortunately, irrigation scheduling by itself is not a particularly effective means of controlling ditch seepage and deep percolation because many irrigators do not have enough physical control over the irrigation flow. Others have limited information on exactly how much water they have because of severe fluctuations in the supply flow rates. Consequently, improved water management is not included in the analysis separately, but is assumed to be an integral part of each structural improvement, the second means of increasing application efficiency.

The variety of structural improvements that might be effective in increasing application efficiency includes lining or piping head and tailwater ditches to eliminate seepage, conversion to alternative irrigation systems to apply water more uniformly and with better control of the application depth, and modification of existing systems (added flow measurement, land leveling, automation, etc.) to improve their efficiencies.

Controlling Head and Tailwater Ditch Seepage

Seepage from head and tailwater ditches in surface irrigation systems can be reduced or eliminated by the lining measures discussed previously. However, with the head ditches, the lined channel must be congruent with the need to divert water at closely spaced intervals. Thus, the piping alternative for seepage reduction in head ditches usually takes the form of gated pipes rather than the continuous sections.

For concrete linings, the salinity cost-effectiveness follows nearly the same argument as presented earlier for concrete lined canals and ditches. As noted previously, small channels and particularly those involved with head ditches and tailwater ditches would not be designed with reduced capacity along the length since these conveyance elements generally must be capable of delivering full capacity along their entire length. The costs for fencing, diversion structures, and safety structures will be smaller for on-farm ditches in most cases than in other conveyance networks. Equation 77 under these conditions would simplify to:

$$\bar{C}_c = 40.1 \cdot Q^{0.56} \cdot L \quad \text{.................................}(99)$$

and Eq. 87 to:

$$S_1 = \Delta S_c \cdot N_d \cdot \Delta SR \cdot WP' \left[\frac{Q_g - Q_p}{Q_g}\right] \cdot 10^{-6} \cdot L \quad \text{.....}(100)$$

Then solving for L in Eq. 100 and substituting the resulting expression into Eq. 99 yields a salinity cost-effectiveness function for on-farm ditches:

$$\bar{C}_c = \frac{40.1 \cdot 10^{6} \cdot Q^{0.56} \cdot S_1}{\Delta S_c \cdot N_d \cdot \Delta SR \cdot WP'} \left[\frac{Q_g}{Q_g - Q_p}\right] \text{...........}(101)$$

It might be worth noting that N_d, the number of days in operation, for head and tailwater ditches must be carefully defined. Both channels are wetted only during irrigations and even then, not over the entire length.

An alternative to concrete head ditch linings is converting to gated pipe. Prices for 6, 8, and 10 inch aluminum gated pipe in 1976 were quite similar to concrete linings for the same purpose. Consequently, a further distinction is probably not justified. The respective choices would be dependent on factors other than salinity control cost-effectiveness.

Irrigation System Conversions

The efficiency and uniformity of surface irrigation methods are dependent primarily upon the infiltration characteristics of the soil. Such systems when not designed and operated in a manner best suited to the soil properties will inherently be inefficient, i.e., have significant deep percolation and seepage losses. Even well designed systems have a comparatively high loss rate because of the large variability in soil properties within the confines of single irrigated fields. One of the most common methods of increasing irrigation application efficiencies is to convert a surface irrigated system into one of the pressurized varieties (sprinkler or drip) in which the amounts applied to the soil are relatively independent of soil properties. In this work, two classes of pressurized systems will be included, sprinkler and drip irrigation.

Sprinkler Irrigation Conversion --

Sprinkler irrigation systems are recommended and used on practically all types of soils and crops with a few limitations due to topography. Flexibility and efficient water control has permitted irrigation of a wider range of soil conditions with sprinklers than most surface water application methods. It has thus allowed irrigation of many thousands of acres (which were previously considered only for dryland farming or as wasteland).

On some saline soils such as in the Imperial Valley of California, sprinklers are recommended for better leaching and crop germination. Sprinklers are especially desirable in soils with high permeability and/or low water holding capacity, although sprinklers can offer distinct advantages over other irrigation methods in dense soils with low permeabilities. In areas where labor is in short supply, sprinklers are among the most economical ways to apply water. In other areas where water costs are high, sprinklers have proven to be economical due to reduced surface runoff. In many cases sprinklers have been shown to increase yields and improve produce quality, particularly for the fresh vegetable and fruit market where color and quality are very important.

Sprinklers, like most physical systems, do have disadvantages. Damage to some crops has been observed when poor quality irrigation water has been applied to the foliage by sprinklers leaving undesirable deposits or coloring on the leaves or fruit of the crop. Sprinklers are also capable of increasing the incidence of certain crop diseases such as fire blight in pears, fungi or foliar bacteria. A major disadvantage of sprinklers is the relatively high cost, especially for solid-set systems. Sprinklers can require large amounts of energy when the water has to be pumped from deep underground aquifers, or when gravity cannot supply sufficient head for operation.

Karmeli (1977) reviewed most of the technical literature and field data pertaining to sprinkler application uniformities before proposing that the most simple and best statistical description of uniformity was a linear regression. The linear function is described as:

$$Y = a + b\,X \qquad (102)$$

where,

Y is a dimensionless precipitation depth equal to $y/\bar{y}$;
X is the fraction of the area receiving less than the applied depth y;
$\underline{a}$, b are linear regression constants;
$\bar{y}$ is the average depth of application; and
y is the depth of application at one fraction of the field area.

The graphical view of Eq. 102 (Figure 14) reveals several interesting characteristics pertaining to sprinkler irrigation efficiencies. An irrigator intending to apply a planned depth or irrigation to his field (Y=1) will discover that 50% of the field will receive less than this average while the other half will receive more. The minimum applied dimensionless depth, Ymin, is the ordinate intercept a, whereas the maximum dimensionless depth, Ymax, is equal to a+b. Karmeli (1977) proposed a uniformity coefficient, UCL, based on Eq. 102 in which UCL is the average deviation from the mean applied depth. Examining Figure 14 indicates the average deviation from Y=1 is 0.25b, thus:

$$UCL = 1 - 0.25b \qquad (103)$$

Use of the UCL as a statistical description offers many advantages over other uniformity criteria. Specifically, the deficiently watered area, the average watered area, the surface watered area, and the respective volumes of water in each of these areas are easily calculated. An examination of Figure 14 reveals that it can also be interpreted as the fractional amount of required water versus the fractional area. Utilizing this concept, it is quite easy to compute the application efficiency. For instance, if Y=1 represents the necessary moisture to refill the soil reservoir (the dimensionless volume actually stored in the root zone) equals $1-(1-Y_{min})^2/2b$ and the volume actually applied is $Y_{min} + b/2$. Application efficiency is therefore,

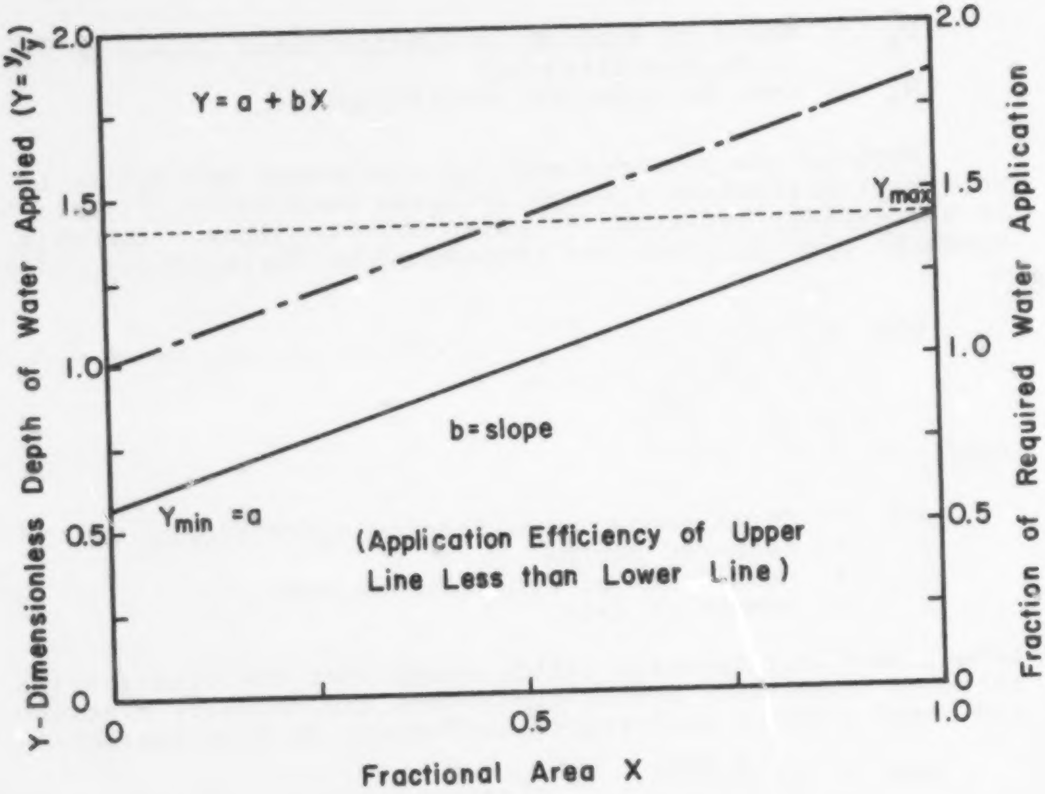


Figure 14. Graphical representation of sprinkler irrigation uniformity analysis (after Karmeli, 1977).

$$AE = \frac{1-(1 - Ymin)^2/2b}{Ymin + b/2} = \frac{1-(1-a)^2/2b}{a+b/2} \quad \text{................(104)}$$

It is also interesting to note that the volume of deep percolation or leaching can be determined from Eq. 104 as follows:

$$D_p = (1-AE)D_a \quad \text{....................................(105)}$$

where,

D_a = depth of applied irrigation water (average), in millimeters; and
D_p = deep percolation in millimeters.

Much of the previous work in evaluation and design of sprinkler irrigation systems utilized uniformity criteria based on statistical deviation in water distributions. The most commonly used approach was introduced by Christiansen (1942),

$$UCC = 1-\frac{\sum_{i=1}^{N} |y_i - \bar{y}|}{N\bar{y}} \quad \text{............................(106)}$$

where,

UCC = Christiansen's uniformity coefficient;
y = measured depth;
$\bar{y}$ = mean depth of application; and
N = number of data.

Later, Hart and Reynolds (1965) found that the distribution patterns under many sprinkler systems are normally distributed and based another uniformity coefficient on this assumption:

$$UCH = 1-\frac{0.798s}{\bar{y}} \quad \text{................................(107)}$$

in which,

UCH = Hart's uniformity coefficient; and
s = standard deviation.

Work by Karmeli (1977) and Hart (1961) established the following relationships between the various uniformity coefficients:

$$UCC = 0.030 + 0.958\ UCH\ (R^2 = 0.888) \quad \text{(108)}$$

$$UCL = 0.011 + 0.985\ UCC\ (R^2 = 0.998) \quad \text{(109)}$$

$$b = 3.956 - 3.940\ UCC \quad \text{(110)}$$

where,

b = the linear slope in Eq. 103 for UCL.

In general, designs which have a UCC value greater than 0.70 and preferably 0.80 are considered satisfactory balances between the costs of increasing uniformity (closer spacings, higher pressure, larger piping) and water losses. Thus, for most sprinkler systems, the value of b in Karmeli's linear analysis would be approximately 0.80, assuming a UCC value also of 0.80. For this value, Eq. 104 can be solved for various values of Ymin to give a relationship for application efficiency:

$$AE = \frac{1 - (1-a)^2/1.6}{a+0.4} \quad \text{(111)}$$

The owner-operator of a sprinkler irrigation system must decide how much, if any, of his field will be supplied with less water than is needed. The result will be a yield decline in the deficient areas but may not be sufficient to offset the costs of additional pumping. If an irrigator supplies the minimum depth area with the desired depth, application efficiency will be about 71%. Willardson, et al. (1977) noted that wind conditions in many areas would allow a Ymin value equal to 0.9 before a significant yield reduction (AE = 76%) would occur. Since most irrigators would not generally allow an appreciable acreage to be water short if water was available, an application efficiency of 76% should represent a typical figure for planning purposes. It might be noted that an upper bound on sprinkler application efficiencies of approximately 90% has been reported (Willardson, et al. 1977).

The salinity reduction achieved by a sprinkler conversion depends on the corresponding decrease in the volume of deep percolation attributed to converting previously surface irrigated lands to sprinkler systems. Lands already sprinkler irrigated would probably not convert to surface systems although a conversion to drip irrigation may be considered. In any event, this analysis assumes that efficiencies among the various types of sprinkler systems are approximately the same and conversion between types is not included. The reduction in deep percolation can be written from Eq. 105:

$$\Delta D_p = (1-\Delta AE)D_a \quad \text{(112)}$$

in which,

ΔD_p = reduction in deep percolation, mm; and

ΔAE = improvement in application efficiency expressed as a fraction.

Then, the salinity reduction attributed to this increased efficiency can be developed from Eq. 78 by replacing V_s by ΔD_p and accounting for evapotranspiration:

$$\Delta S_{AE} = \frac{\Delta S_c}{(1-AE')} \cdot \Delta D_p \cdot \left[\frac{Q_g - Q_p}{Q_g}\right] 10^{-5} \quad \text{.............(113)}$$

in which,

ΔS_{AE} = reduction in salt loading due to improved application efficiencies, metric tons/year/ha; and

AE' = application efficiency under improved conditions (expressed as a fraction).

It should be noted that if sprinkler conversion results in the elimination of field head and tailwater ditches, the corresponding salinity reduction needs to be added to Eq. 113.

In addition to permanent solid-set sprinklers, side-roll wheel-move sprinkler and center pivot sprinklers, there are several other commerically available sprinkler systems which could potentially be used in an area. These include hand-move portable systems, traveler or "big gun", and tow-line systems.

Hand-move portable systems were the first type of sprinkler systems developed and still enjoy wide popularity. They are usable in any situation where any other types of sprinklers can be used. As the name implies, the systems are moved from set to set by hand labor. The mainline for these systems may be either buried and permanent, or may also be portable.

Many means have been devised to circunvent the labor problem of hand-move portable systems, one of which is the center pivot. This sprinkler system rotates about a central point (pivot) in a large circular or ellipical pattern and most commonly covers 55 to 61 hectares (135 to 150 acres) per system. The water is supplied through the pivot to sprinklers variably spaced along the pipe. The water application rates on a center-pivot system increases from the center to the end of the system due to the increase in areal coverage. These systems have several disadvantages. As one example, the corners of a square field are not irrigated, unless by other systems. In addition, the water distribution is often poorer than other systems.

On some heavy soils where the application rate is higher than the infiltration rate, erosion may result from the surface, and traction problems can affect the machine operations.

Traveler or "big gun" systems are usually limited to soils with high infiltration rates since the sprinkler head is essentially a big gun shooting a large volume of water up to 60 meters (200 feet) or more. These sprinklers can move down a lane in the field with a trailer arrangement pulling a long flexible hose while irrigating, hence the name traveler. "Big-gun" sprinklers can also be mounted on permanent towers, which then becomes a high volume permanent solid-set system.

There are several variations on the traveler design including a traveling boom type of system which uses several smaller sprinklers mounted on a large boom to cover approximately the same area. Traveler systems have many of the same disadvantages of center-pivot systems although instead of losing land at the corners of a field, land is lost in the travel lanes every 46 to 122 meters (150 to 400 feet).

Another type of sprinkler system quite similar to the side-roll wheel-move concept is the end-tow or tow line. As the name indicates, the system is mounted on skids or wheels, but is towed from the end to the next set by a tractor or other vehicle.

A summary of costs for various sprinkler irrigation systems is shown in Figure 15. Each of the cost functions have the form:

$$y = a' + b'/x \quad \ldots\ldots (114)$$

where,

y = relative capital cost (\$1800/ha = 1.0);
a', b' = regression coefficients; and
x = relative field size in hectares (25 ha = 1.0).

The cost-effectiveness of a sprinkler irrigation conversion is then determined by dividing Eq. 114 by Eq. 113.

Trickle Irrigation Conversion --

Trickle (or drip) irrigation has emerged during the last 5-10 years as a highly cost-effective method of applying irrigation water directly to crop root zones. The basic concept is to deliver water to each plant or group of plants on a frequent, low moisture tension basis. Tests from all over the world have demonstrated significant yield increases, water savings, and reduced labor requirements from trickle irrigation systems. Until now, however, the development of the system

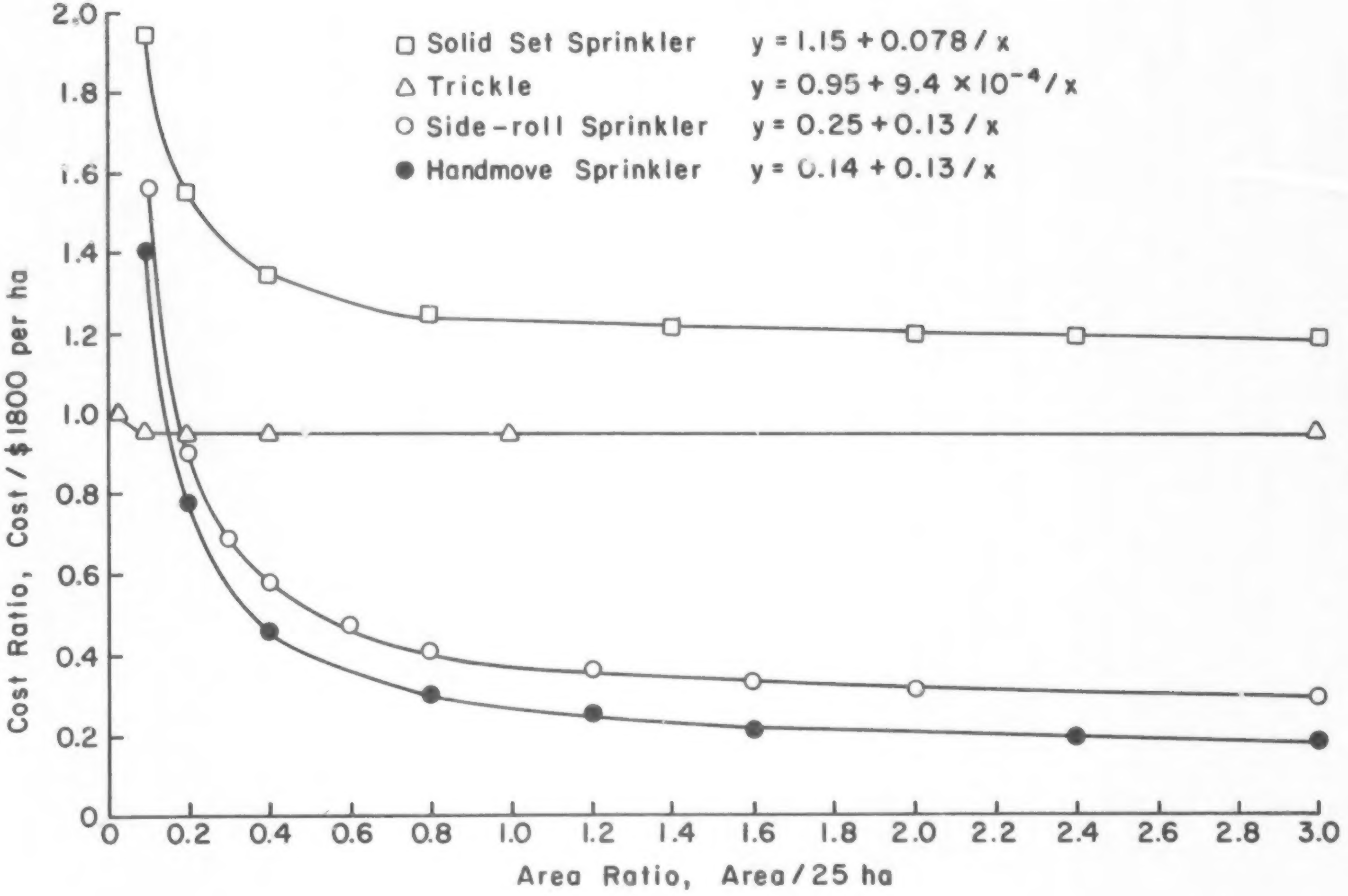


Figure 15. Dimensionless cost functions for various types of pressurized irrigation systems.

hardware has been somewhat inadequate in meeting the needs encountered in areas where the water contains substantial quantities of sediment or organic materials. Today, the trickle irrigation technology is well developed and tested. The costs of trickle irrigation systems are shown in Figure 15 (Geohring, 1976). This work was based on orchard systems where the costs would be substantially lower than for agronomic or vegetable crop applications.

A detailed analysis of trickle irrigation uniformities and efficiencies is given by Keller and Karmeli (1975) and by Goldberg, et al. (1976). The standard design practice is to provide for a 10% or less leaching fraction. Application efficiencies, therefore, would be designed at the 90% and above level. The existing literature as summarized by Smith and Walker (1975) indicates that this level of application is generally achieved in field situations. Consequently, application efficiencies can be assumed at 90% unless otherwise indicated. The resulting cost-effectiveness for salinity control can then be determined as described in the previous paragraphs.

Automation of Surface Systems --

Water and energy savings are often noted as limiting resources in irrigated agriculture, but neither has produced the changes in the irrigation industry that are attributable to labor savings. In fact, water and energy savings are generally achieved as a result of more capital intensive, automated irrigation systems because irrigation efficiencies are increased. Automation has, therefore, become one of the most consistent trends in irrigation.

A completely automatic surface irrigation system would not only sense the need for an irrigation, but also route the water from the source to the field, irrigate the field effectively, and turn off and reset the system for future operations. Most of the automation incorporated in irrigation systems has been limited to control of the volume of water applied to the field. In surface irrigation, the principal use of automation has been associated with cutback irrigation in which the field is watered thoroughly with an initially high flow rate to wet the field and then the flow is reduced to allow for adequate intake opportunity time without excessive tailwater.

The impact that automation might have on salinity which is primarily a subsurface problem is limited to controlling the duration of irrigations so that the depth of infiltration closely approximates the soil moisture storage capacity in the root zone. Thus, automation is primarily a substitute for labor during irrigations, in fact, up to 92% (Humphreys, 1971). Irrigation efficiencies have been reported as high as 95% under automated systems (Worstell, 1975) but are generally found to be 75-85% (Somerhalder, 1958; Fischbach and Somerhalder, 1971; Humphreys, 1971; and Evans, 1977). Costs have to be determined for a variety of systems although no general estimating analyses have been completed. For most purposes, automation of surface irrigation systems can be estimated to be 1.5 to 2 times the costs of concrete linings or gated pipe (Worstell, 1975); Evans, 1977).

Evaluation of Existing Efficiencies

The increase in application efficiency achieved by either a conversion to another method of irrigation or improvements in the existing system depends also upon the efficiency of the existing irrigation practices. This difference in application efficiency is very important to the cost-effectiveness relationship for managing the quality of irrigation return flows. For instance, if furrow irrigation application efficiencies are nearly those of a proposed sprinkler system, the implementation of the conversion would mean high costs with low salinity reductions.

Evaluation of existing surface irrigation application efficiencies is often a difficult task unless some proven mathematical approaches are applied. The primary element in surface irrigation evaluations is definition of soil intake or infiltration rates. Many empirical equations have been proposed, but the most commonly employed is the relationship introduced by Kostiakov (1932):

$$i = at^b \qquad (115)$$

where,

i = infiltration rate in cm/min;
t = interval since infiltration began, min; and
a,b = empirical regression coefficients.

Integrating Eq. 115 over the irrigation interval yields:

$$I = \frac{a}{b+1} t^{b+1} = At^B \qquad (116)$$

in which,

I = cumulative soil infiltration, cm.

Because the intake opportunity time varies in a field due to the time required for water to reach a point, the infiltration depth over a field's length will also vary. A commonly employed function expressing the relationship between the advance rate and time is:

$$x = pt_x^r \tag{117}$$

where,

x = distance along the flow path in cm;
t_x = time to advance x centimeters, min; and
p, r = empirical regression coefficients.

Actually, the parameter r can be very well approximated without field data if the infiltration exponent, B, is known (Fok and Bishop, 1965):

$$r = \exp(-0.6B) \tag{118}$$

Generally, however, r is determined by field data enroute to defining B. If Eq. 117 is adequate, the value of r can be determined by knowing the advance time to any two points along the field. For instance,

$$r = 0.69/\ln T \tag{119}$$

in which,

$$T = t_L/t_{0.5L} \tag{120}$$

where,

$t_{0.5L}$ = time necessary to advance one-half the field length, L; and
t_L = time necessary to advance the entire field length.

It can be seen that with very simple advance data, the exponent in Eq. 117 can be determined, thereby also dictating p when one point is observed. It should be noted however, the parameter p encompasses surface roughness and flow rate and will change from irrigation to irrigation.

The volume (V_ℓ) of water infiltrated into the wetted furrow length at any time, t_ℓ, is determined by:

$$V_\ell = 10^{-2} \cdot \int_0^x I\,dx = 10^{-2} \cdot A \int_0^x (t_\ell - t_x)^B dx \tag{121}$$

Since $dx = (\partial x/\partial t_x) \cdot dt_x$, the limits of integration can be expressed in limits of time with the aid of Eq. 117:

$$V_\ell = A \cdot p \cdot r \cdot 10^{-2} \cdot \int_0^{t_\ell} (t_\ell - t_x)^B \cdot t_x^{r-1} dt \tag{122}$$

Noting also that if $t = t_x . t_\ell$, $(0 \leq t \leq 1)$, is substituted into Eq. 120, a definite integral results:

$$V_\ell = A \cdot p \cdot r \cdot t_\ell^{B+r} \cdot 10^{-2} \cdot \int_o^1 (1-t)^B t^{r-1} dt \text{ .... (123)}$$

which has the following solution,

$$V_\ell = A \cdot p \cdot t_\ell^{B+r} \beta(B+1, r+1) \cdot 10^{-2} \text{ .... (124)}$$

where,

β = beta function.

Christiansen, et al. (1966) showed that the beta function could be closely approximated for conditions found in surface irrigation by the expression:

$$\beta(B+1, r+1) = (b - b \cdot r + 2) \Big/ \Big[(b+2)(r+1)\Big] \text{ .... (125)}$$

To evaluate the parameters needed to utilize Eq. 124, a mass balance approach may be taken for a series of lengths such that:

$$V_i = \frac{V_q - V_s}{w} \text{ .... (126)}$$

where,

V_q = volume (constant flow rate) introduced into the furrow at time t_1, cm ^{3}/cm;
V_s = volume in surface storage at time t_i, cm ^{3}/cm;
w = furrow spacing (or unit width for borders), cm; and
V_i = value of V_ℓ for various advance distances, x_i.

Generally, V_q is measured at the field inlet leaving V_s to be determined. Wilke and Smerdon (1965) proposed that the average cross-sectional area of flow be described by the relationship:

$$C = M d_o^N \text{ .... (127)}$$

in which,

C = average cross-sectional area, cm^2;
d_o = normal depth at field inlet, cm;
M = 8.59 for furrows and 1 for borders; and
N = 1.67 for furrows and 1 for borders.

Both M and N can be evaluated by furrow cross-sectional measurements. The normal depth was based on Manning's Equation (n = 0.047):

$$d_o = 0.60 \left[Q/s_o^{0.5}\right]^{0.4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (128)$$

in which,

Q = furrow flow rate or border unit flow rate, ℓ/min; and
s_o = field slope in percent.

By substituting the right-hand side of Eq. 124 for V_i in Eq. 126 (t_ℓ becomes t_i) and noting that the product of C in Eq. 127 and the advance distance x_i equals V_s, Eq. 126 becomes after simplifying and correcting for units:

$$A \cdot \beta(B+1,r+1)t_i^{B+r} = \frac{V_q - x_i C \cdot 10^{-4}}{w \cdot p \cdot 10^{-2}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (129)$$

A logrithmic plot of the right side of Eq. 129 against time, t_i, will yield a straight line of slope B+r from which each of the unknowns in the infiltration equation (A,B) can be determined.

Application efficiencies can be determined from the preceding analysis by defining the average depth of moisture needed to refill the crop root zone, D, in cm. Then from Eq. 116, the time necessary to infiltrate the desired depth is:

$$t_D = (D/A)^{1/B} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (130)$$

where t_D is time in minutes water must infiltrate at a specific point to replace the depth D in the root zone. Three cases for the furrow irrigation regime may be detailed: (1) the under-irrigated case where the lower reaches are not completely refilled (Figure 16); (2) the case where the minimum irrigated area is refilled (Figure 17); and (3) the general over-irrigated case (Figure 18).

The earlier discussion regarding the infiltration evaluation covered only the advance phase of irrigation. Actually, the analysis is valid for periods longer than this by assuming that the furrow extends indefinitely. Thus, if the total time water is applied to the furrow (set time), is again represented by t_ℓ, $t_\ell > t_L$, the length of advance predicted by Eq. 117 would be:

$$\ell = p t_\ell^{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (131)$$

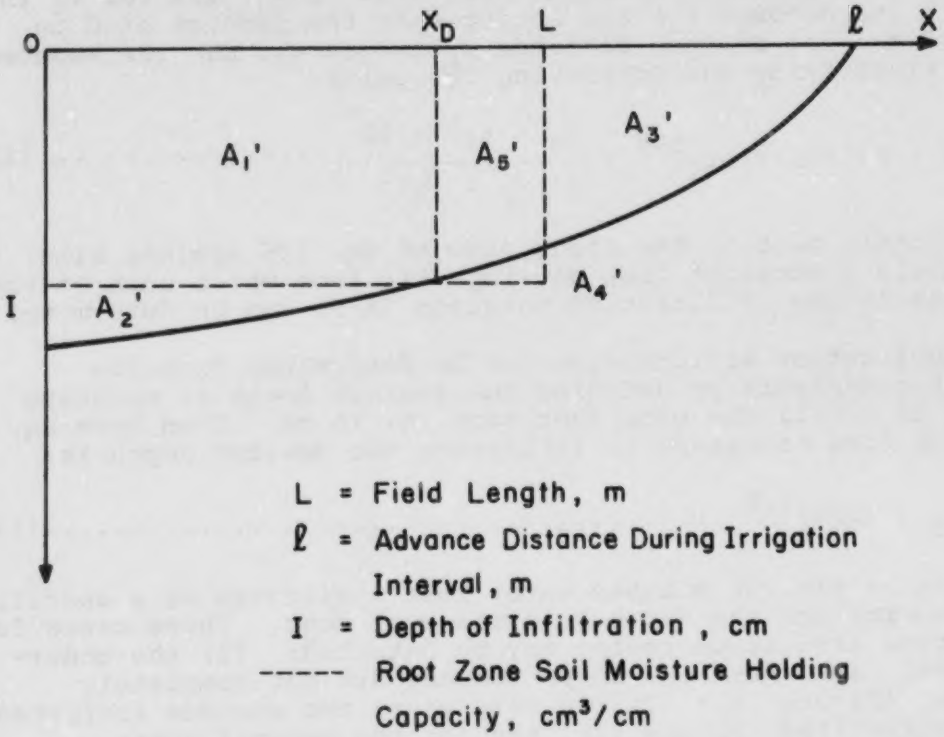

O
X_D
L
ℓ
X
A_1'
A_5'
A_3'
A_4'
A_2'
I
L = Field Length, m
ℓ = Advance Distance During Irrigation Interval, m
I = Depth of Infiltration , cm
----- Root Zone Soil Moisture Holding Capacity, cm³/cm

Figure 16. Definition sketch of surface irrigation application uniformity in the case where part of the field is under-irrigated.

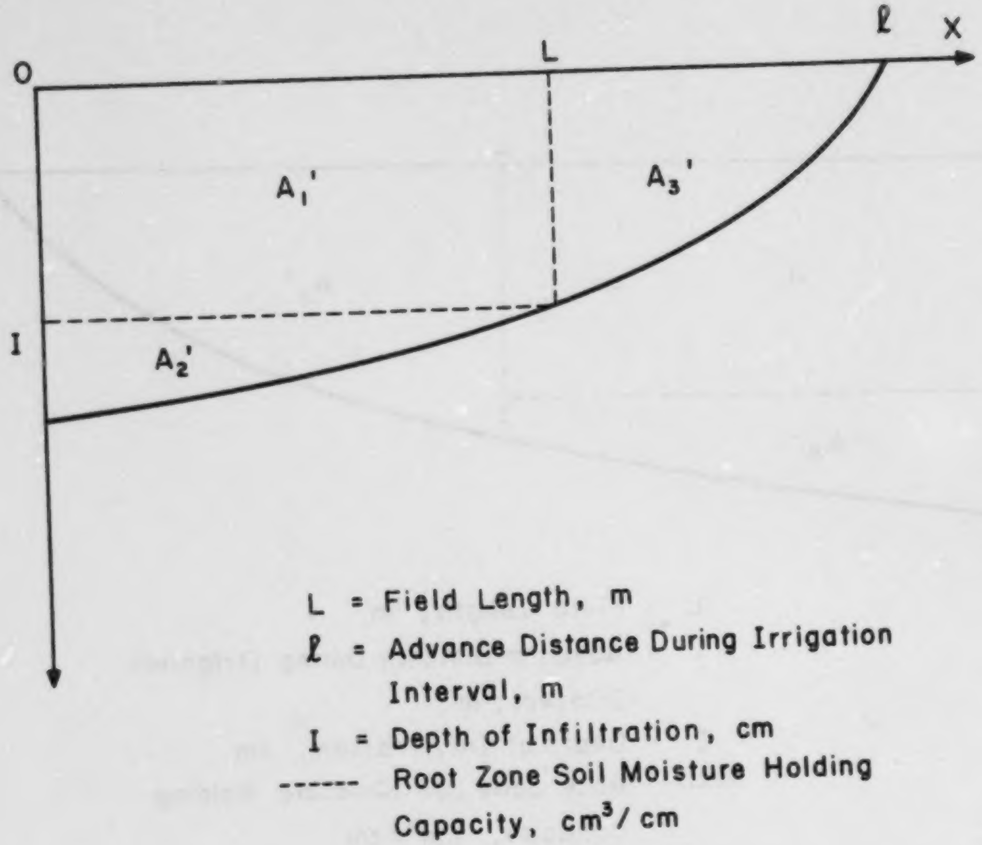

O
L
ℓ
X
A₁'
A₃'
I
A₂'
L = Field Length, m
ℓ = Advance Distance During Irrigation Interval, m
I = Depth of Infiltration, cm
------ Root Zone Soil Moisture Holding Capacity, cm³/cm

Figure 17. Definition sketch of surface irrigation application uniformity in the case of zero under-irrigation.

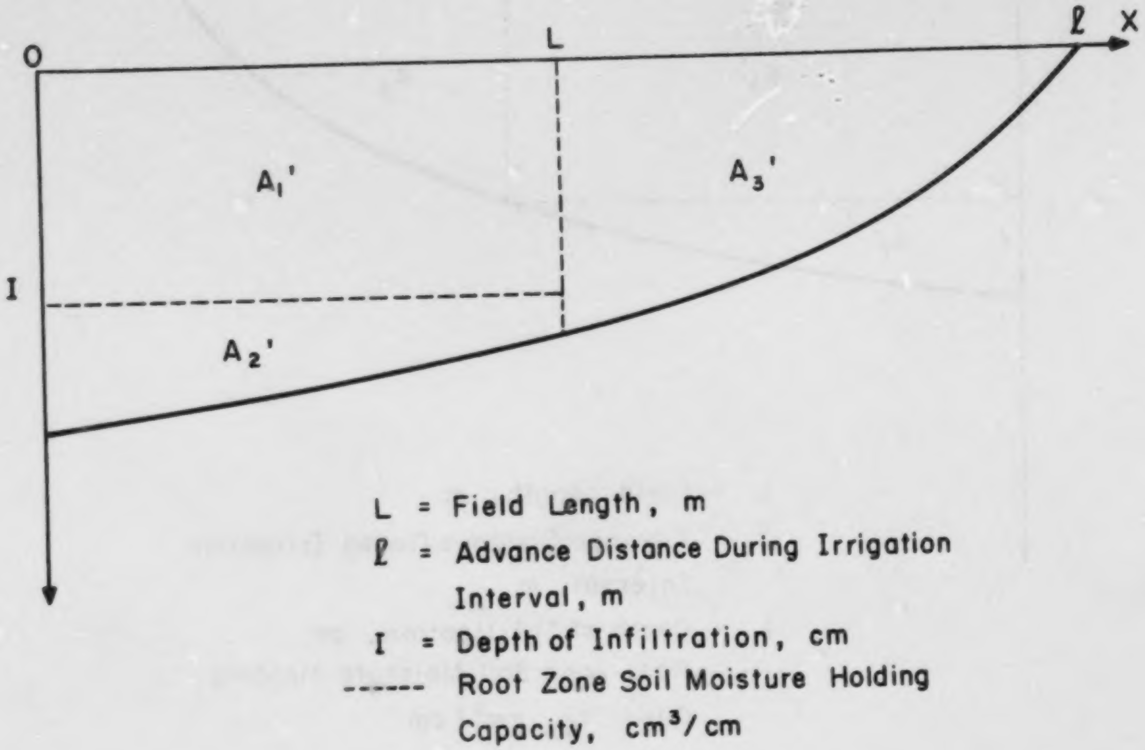

O
L
ℓ
X
A₁'
A₃'
I
A₂'
L = Field Length, m
ℓ = Advance Distance During Irrigation Interval, m
I = Depth of Infiltration, cm
------ Root Zone Soil Moisture Holding Capacity, cm³/cm

Figure 18. Sketch of surface irrigation uniformity under conditions of significant over-irrigation.

where ℓ is the total "equivalent" field length, m. The volume of infiltration over the length ℓ, V_ℓ is determined from Eq. 124.

Referring to Figures 16, 17, and 18 again, it is seen that in order to compute the volume of deep percolation and tailwater, Eq. 123 must actually be solved for various fractions of t_ℓ. Specifically, for any time t' such that $0<t'<t_\ell$, the resulting equation is:

$$V' = 10^{-2}\cdot A\cdot p\cdot r\cdot t_\ell^{B+r} \int_0^R (1-t)^B t^{r-1} dt \text{ .... (132)}$$

where,

V' = the volume of water infiltrated over a segment of the "equivalent" furrow, m^3/m; and

$R = t'/t_\ell$.

Since Eq. 132 does not have an exact solution, a numerical integration technique was utilized by Gerards (1977) to compute the integral part for a wide range of values for B, R, and r. A multiple correlation analysis found that the solutions could be related to values of R and B as follows:

$$M_R' = r \int_0^R (1-t)^B\ t^{r-1}\ dt = eR^f \text{ .... (133)}$$

where,

$$e = 0.9598 \exp(-0.3383B) \text{ .... (134)}$$

and,

$$f = 1.0170 \exp(-0.9763B) \text{ .... (135)}$$

The maximum error (e_M) between Eq. 133 and the 800 increment numerical solution was $-1.7\% < e_M < 2.7\%$. It should also be noted that the parameter r was eliminated by assuming that the relationship in Eq. 118 was valid.

It now is possible to utilize the preceding analysis to compute the irrigation efficiency for a furrow system. To do this, two specific subsets of irrigation efficiency are defined. The first, application efficiency, AE, is:

$$AE = \frac{\text{root zone storage}}{\text{total infiltrated volume}}$$

$$= \frac{A_1' + A_5'}{A_1' + A_5' + A_2'} \times 100 \tag{136}$$

The second, field efficiency, FE, is:

$$FE = \frac{\text{root zone storage}}{\text{total field deliveries}}$$

$$= \frac{A_1' + A_5'}{A_1' + A_2' + A_3' + A_5'} \times 100 \tag{137}$$

Note that where the least irrigated areas along the furrow are refilled, A_4' and A_5' are equal to zero.

Values for the respective segments of the infiltrated and runoff volumes along a furrow are determined using the solutions to Eq. 132 as follows:

$$A_1' = X_D D,\ X_D \leq L \tag{138}$$

$$A_2' = 10^{-2} \cdot A \cdot p \cdot t_\ell^{B+r} \cdot M'_{R_D} - A_1' \tag{139}$$

$$A_5' = 10^{-2} \cdot A \cdot p \cdot t_\ell^{B+r} \cdot M'_{R_L} - A_1' - A_2',\ X_D \leq L \tag{140}$$

$$A_5' = 0,\ X_D \geq L \tag{141}$$

$$A_4' = L \cdot D - A_1' - A_5',\ X_D \leq L \tag{142}$$

$$A_4' = 0,\ X_D \geq L \tag{143}$$

$$A_3' = 10^{-2} \cdot A \cdot p \cdot t_\ell^{B+r} \left[\frac{b - br + 2}{(b+2)(r+1)}\right] - A_1' - A_2' - A_5' \tag{144}$$

where,

$$M'_{R_D} = r \int_0^{R_D} (1-t)^B\, t^{r-1} dt,\ R_D = t_D/t_\ell \tag{145}$$

and,

$$M'_{R_L} = r \int_0^{R_L} (1-t)^B t^{r-1} dt,\ R_L = t_L/t_\ell \tag{146}$$

SECTION 7

OPTIMIZING DESALINATION-AGRICULTURAL SALINITY CONTROL STRATEGIES

INTRODUCTION

The mathematical procedure outline earlier describing the selection of optimal salinity control strategies for four subdivisions within a river basin might be more clearly illustrated through an application to a case study. Walker (1976) and Walker, et al. (1977) presented a similar analysis using preliminary results of this project and that reported by Evans, et al. (1978a, 1978b) and Walker, et al. (1978). The Grand Valley of western Colorado is the principal focus of the work cited above. Its popularity as a location of field scale research into the Colorado River salinity problem in recent years provides a convenient setting for the developement of an analysis such as contained in this report because of a comparatively large data base. The report by Walker, et al. (1978) contains the results included in the following paragraphs and an evaluation of the sensitivity of the results to the input data and assumptions. This report is intended as a presentation of the analytical development, and therefore, will not consider the various model sensitivities under Grand Valley conditons. The cost-effectiveness parameters selected for the calculations contained in the report have been taken from the references cited here with the exception of the on-farm improvements relationships. The results reported by Walker, et al. (1977) were based on very preliminary estimates of the on-farm costs and associated salinity reductions, and tend to be towards the conservative side of the range of possible values. Results presented herein will encompass the high range and will therefore indicated some differences in the optimal practices in the valley.

It will become apparent to the reader that an interesting evolution has occurred in the study of agricultural pollution problems. At the earliest stages most of the investigative efforts are devoted to problem identification in small "representative" areas in a system. Collected data are detailed in both spatial and time references and simualtion models are sophisticated treatments of the complex physical-chemical-biological irrigation system. These studies indicate the interrelationship between the natural and operational system, thus, pointing out the factors of most impact. Attention then is diverted to extending the "laboratory" studies to the full scale of the irrigated valley or subbasin. Parameters become lumped through averaging and time resolutions are aggregated into weekly or monthly events. Models become input-output devices using mass balance as the main verification or calibration criterion, but they absolutely assume that their inherent simplifying assumptions are congruent with the specific nature of the real system as understood from the detailed modeling evaluations. And finally, the question is asked, "what should be done to improve the quality of the return flows?" Input becomes "single-valued", "long-term average", and annual in nature. Models become management or optimizational types rather than simulation tools, but must again conform to the essential boundary conditions identified by the more detailed analysis.

In this section, the application of the optimizational modeling approach is illustrated for two levels of a four-level problem. And although the results do not include sensitivity analysis due to the scope of the report, it should be noted that the integration of the studies mentioned in the previous paragraphs tends to mask the spatail variability inherent in the real system. Consequently, sensitivity analysis becomes the only effective method of insuring that this variability is considered.

DESCRIPTION OF THE CASE STUDY AREA

In the mid-1960's, a concerted effort was undertaken to identify the sources of salinity in the Colorado River Basin. The Federal Water Pollution Control Administration, then within the Interior Department, utilized U.S. Geological Survey stream gaging data as well as an extensive water quality sampling program to identify the major salt contributors in the basin (U.S. Environmental Protection Agency, 1971). The Grand Valley in western Colorado was described as one of the largest agricultural sources of salinity (about 18% of the total Upper Basin's agriculturally related contribution), and it subsequently became the site for the first studies to evaluate field-scale salinity control measures.

The early studies identified the Grand Valley as a major problem area by mass balancing water and salt flows into and out of the valley region. Similar investigations by Iorns et al. (1965) and Hyatt et al. (1970) produced supportive results, although a substantial variability in the specific nature of the Grand Valley salinity problem emerged. Since that time, a great many individual calculations describing the valley salt loading and the respective components have been made, but very little agreement existed until early 1977 when most studies were completed. Although some variability still exists among the various investigative groups, the differences are sufficiently close that they become relatively unimportant in evaluating optimal salinity control policies in the valley.

Problem Identification

The procedures for delineating the water-salt flow system in an irrigated area are collectively termed hydro-salinity budgeting, or hydro-salinity modeling, since computers are generally needed to handle the large number of necessary calculations. In the Grand Valley, the composition of this system has been extensively investigated at various levels of sophistication. As the salinity investigations were continued, refinements in the valley's basin-wide impact have been made and verified. Interestingly, the research evolution in the Grand Valley case study suggests a fairly sound approach for other areas as well.

The problem of remedying an irrigation return flow causing detrimental water quality deterioration can be divided into four logical steps. First, the magnitude of the problem and the downstream consequences must be identified in relation to the irrigated area's individual contribution to the problem. In this way, the most important areas can be delineated for further consideration, thereby making the most cost-effective use of available personnel and funding resources. As noted previously, this step led to the exhaustive efforts in the Grand Valley that this case study reports. Next, the components of the problem must be segregated. In most large areas, the costs of studying the entire system are prohibitive, so smaller "sampling" studies are conducted from which projections are made to predict the behavior of the entire area. The third step is to evaluate management alternatives on a prototype scale in order to assess their cost-effectiveness and develop a sensitivity about the capability for implementing such technologies. And finally, if the measures which can be applied are effective in reducing salinity and are economically feasible, the final step is the actual application of the technology to solving the water quality problem.

The Grand Valley was identified as an important agricultural source of salinity in the Colorado River Basin through a series of analyses involving mass balance of the valley inflows and outflows. Iorns et al. (1965) evaluated stream gaging records for the 1914 to 1957 period, concluding that net salt loading (salt pickup) from irrigation in the valley ranged from about 450,000 to 800,000 metric tons annually. This range of numbers has been generated independently by Hyatt et al. (1970), Skogerboe and Walker (1972), Westesen (1975), and the U.S. Geological Survey (1976). More recent consideration of data by the writer and others indicates a long-term salt pickup rate between 600,000 to 700,000 metric tons/year. This figure is now generally accepted by the various research groups and action agencies involved with Grand Valley salinity investigations.

The fact that the valley's salinity contribution has been such a disputed figure over the last five years exemplifies the importance of establishing the total valley contribution. In areas like the Grand Valley where the total valley impact is only 5-8% of the river inflows or outflows, the impact of irrigation must be established using statistical analyses of the available data. However, the natural variability can cause serious errors in conclusions regarding salt pickup if not tempered by other data. For example, a major problem in early investigations was deciding how much of the inflow-outflow differences was due to natural runoff from the surrounding watershed. Because of the meager precipitation locally, the writers assumed the natural salt contribution would be negligible. This conclusion was later substantiated partially by Elkin (1976) who estimated an upward limit for the natural contribution of about 10% of agricultural figures.

Segregating the Irrigation Return Flow System

In the Grand Valley, as in numerous other irrigated areas, water is supplied to the cropland in a canal, ditch, and lateral conveyance system. Water is diverted from the Colorado and Gunnison Rivers into three major canals: (1) the Government Highline Canal; (2) the Grand Valley Canal; and (3) the Redlands Power Canal. These large canals in turn supply the smaller canals and ditches as follows:

Government Highline Canal	Grand Valley Canal	Redlands Power Canal
Stub	G.V. Mainline	Redlands #1
Price	G.V. Highline	Redlands #2
Orchard Mesa Power	Mesa County	
Orchard Mesa #1	Kiefer Extension	
Orchard Mesa #2	Independent Ranchmen's	

A description of the hydraulic characteristics of these canals and ditches is given in Table 11, based on information provided by the Bureau of Reclamation. From the canals and ditches, water is diverted into the small, largely earth ditches leading to the individual fields. This lateral system of approximately 600 kilometers of ditch carrying from 0.06 - 1.0 m^3/sec. A frequency distribution of the lateral lengths based on data provided by the Bureau of Reclamation indicated that the average length is about 400 meters, with an average capacity of about 0.10 m^3/sec.

TABLE 11. HYDRAULIC CHARACTERISTICS OF THE GRAND VALLEY CANAL AND DITCH SYSTEM

Name	Length (km)	Initial Capacity (m^3/sec)	Terminal Capacity (m^3/sec)	Initial Perimeter (m)
Government Highline Canal	73.70	16.99	0.71	19.19
Grand Valley Canal	19.80	18.41	14.16	16.67
Grand Valley Mainline	21.70	7.08	0.71	13.86
Grand Valley Highline	37.00	8.50	3.96	12.62
Kiefer Extension of Grand Valley	24.50	3.96	0.71	7.25
Mesa County Ditch	4.00	1.13	0.06	6.67
Independent Ranchmen's Ditch	17.40	1.98	0.85	3.17
Price Ditch	9.50	2.83	0.28	7.27
Stub Ditch	11.30	0.85	0.11	2.94
Orchard Mesa Power Canal	3.90	24.07	24.07	18.20
Orchard Mesa #1 Canal	24.10	3.12	0.17	6.46
Orchard Mesa #2 Canal	26.10	1.98	0.17	3.58
Redlands Power Canal	2.90	24.07	24.07	16.88
Redlands #1 & #2 Canals	10.80	1.70	0.06	3.95

Nearly all farmers in the valley apply water using the furrow irrigation method. The Soil Conservation Service (SCS) inventory of the valley's irrigation system indicates over 9,000 individual fields in the valley having a wide range of widths, slopes and lengths. The typical field is 140 meters wide, 160 meters long, with a slope (toward the south generally) of 1.125%. A frequency distribution of field acreages showed the typical field encompassing a little more than 2 hectares. Calculating the length of unlined field head ditches based on the SCS data indicates a total length of 1300 kilometers.

Irrigation water is applied to approximately 25,000 hectares during the course of a normal irrigation season (Walker and Skogerboe, 1971). This acreage has been substantiated by the recent SCS inventory and generally accepted by the other agencies. A graphical breakdown of the acreage and miscellaneous land use in the valley is given in Figure 19.

Based upon lysimeter data reported by Walker et al. (1976) the weighted average consumptive use demand by the irrigated portion of the area equals about 0.745 meters per season. This breakdown of the individual consumptive uses in the valley is given in Table 12.

The irrigation return flow system in the valley may be divided according to whether or not the return flows are surface or subsurface flows. Surface flows occur as either field

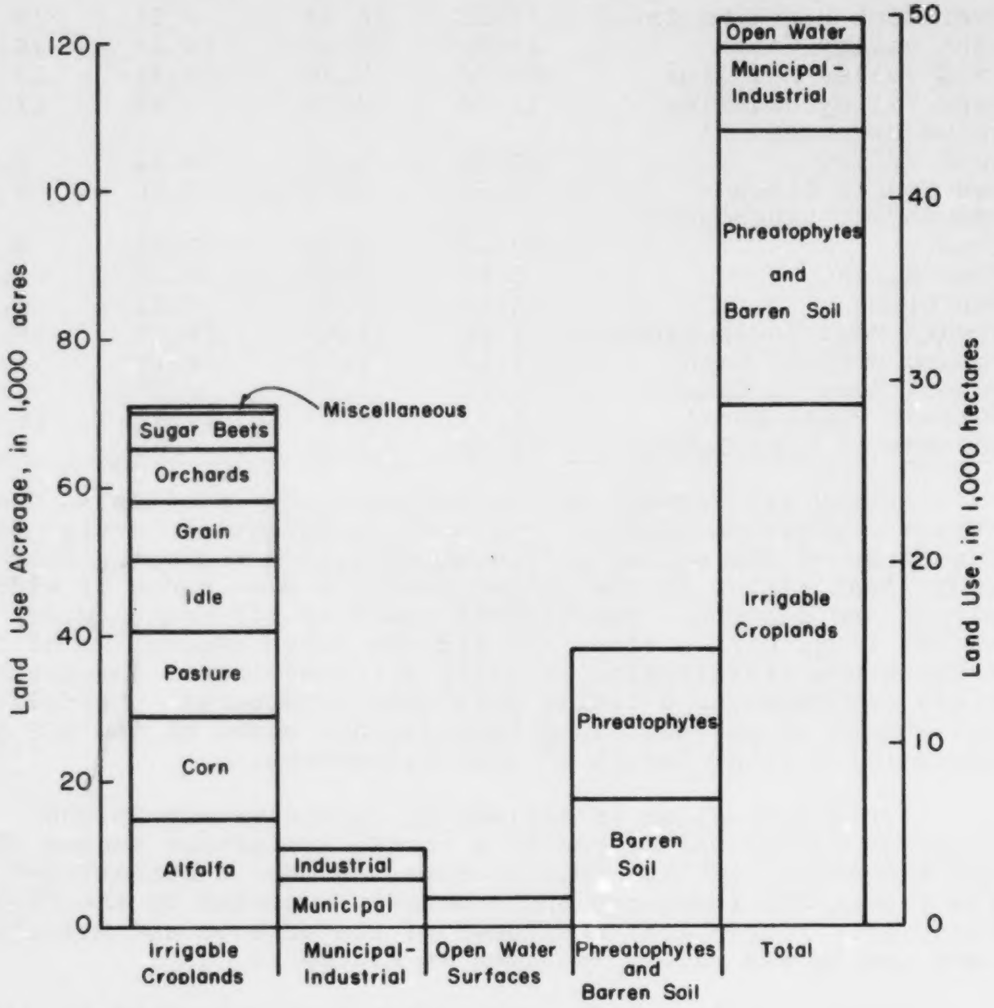


Figure 19. Land use in the Grand Valley.

TABLE 12. CONSUMPTIVE USE ESTIMATED FOR THE GRAND VALLEY

Consumptive Use	Volume in ha-m	Depth in Meters
Open water surface evaporation and phreatophyte use[1]	3,450	0.138
Open water surface evaporation and phreatophyte use[2]	8,400	0.336
Cropland	18,600	0.745
TOTAL	30,450	1.219

[1] adjacent to river
[2] along canals and drains
[3] assumed area of 25,000 ha

tailwater or canal, ditch, and lateral spillage. Subsurface flows include canal and ditch seepage, lateral seepage, and deep percolation from on-farm applications (deep percolation in this sense to include head ditch and tailwater ditch seepage).

Canal and Ditch Seepage --

Since the early 1950's, five major seepage investigations on the valley's major canals and ditches have been conducted (Skogerboe and Walker, 1972 and Duke et al. (1976). Although seepage rates have been noted over a wide range, some representative rates are presented for the fourteen canal systems in Table 13. Substitution of the values in Table 13 into the equations of Section 6 yields a seepage volume for each canal and ditch (Table 13). In the Grand Valley, the canal seepage is estimated to be approximately 3,700 ha-m per year.

Lateral Seepage --

Tests reported by Skogerboe and Walker (1972) and Duke et al. (1976) indicate seepage losses from the small ditches comprising the lateral system probably average about 8 to 9 ha-m/km/year in the Grand Valley. Thus, for the 600 km of small laterals, the total seepage losses are approximately 5,300 ha-m annually. Combined lateral and canal seepage is, therefore, approximately 9,000 ha-m annually.

TABLE 13. SEEPAGE DATA FOR THE FOURTEEN MAJOR CANAL SYSTEMS IN THE GRAND VALLEY.

Name of Canal or Ditch	Days in Operation	b	Seepage Rate m^3/m^2/day	Seepage Volume ha-m
Government Highline	214	0.80	0.091	1652.53
Grand Valley	214	0.17	0.045	290.84
Grand Valley Mainline	214	0.69	0.061	257.16
Grand Valley Highline	214	0.29	0.061	521.16
Kiefer Extension	214	0.60	0.061	162.31
Mesa County	214	0.64	0.061	23.68
Independent Ranchmen's	214	0.31	0.061	60.84
Price	214	0.65	0.061	60.86
Stub	214	0.44	0.061	33.83
Orchard Mesa Power	365	0.001	0.076	196.80
Orchard Mesa #1	214	0.72	0.076	162.05
Orchard Mesa #2	214	0.62	0.076	104.86
Redlands Power	365	0.001	0.065	116.08
Redlands 1 & 2	214	0.67	0.137	83.17
				3726.17

On-Farm Deep Percolation --

Numerous studies in recent years have attempted to quantify deep percolation from on-farm water use. Skogerboe et al. (1974a, 1974b) estimated these losses (including head ditch and tailwater ditch seepage) to be about 0.30 ha-m/ha. Duke et al. (1976) estimated these losses, independent of ditch seepage, to be 0.15 ha-m/ha. Minutes of the Grand Valley Salinity Coordination Committee show on-farm ditch seepage to be 0.12 ha-m/ha (Kruse, 1977). Combining the figures given by Duke, et al. (1976) with Kruse (1977) gives a total on-farm subsurface loss of 0.27 ha-m/ha. Given the large number of fields tested by various investigators, total on-farm losses are probably about 7,500 ha-m/per year.

Canal Spillage and Field Tailwater --

The operational wastes and field tailwater are difficult to define because, first, they do not generally create problems associated with salinity degradation, and second, data regarding these flows are sparse. Skogerboe et al. (1976b) listed field tailwater as 43% of field applications, whereas Duke, et al. (1976) reported estimates of canal spillage or administrative wastes which were 18% and 35%, respectively. Estimates of spillage and tailwater by the Bureau of Reclamation were slightly smaller than the author's estimate. Using the 43% figure for field tailwater and the 18% figure for canal spillage yields about 37,000 ha-m per year field tailwater and spillage.

Aggregating the data presented previously with inflow-outflow records in the vicinity of Grand Valley gives a clear picture of how the irrigation system relates to the overall hydrology (Figure 20). The flow diagram is particularly helpful in visualizing the relative magnitude of the irrigation return flows from the agricultural area.

Identifying the Salinity Contribution

The salinity contribution of the Grand Valley hydro-salinity system can be developed in a number of ways. For example, if the annual salt pickup is divided by the volume of groundwater return flow (630,000 tons/8100 ha-m), the average concentration of the return flow can be determined (7,800 m/ℓ). Data reported by Skogerboe and Walker (1972) indicated an average groundwater salinity of 8,000 to 10,000 mg/ℓ (average of 8,700 mg/ℓ) if the irrigation water salinity is 500-1,000 mg/ℓ.

The U. S. Geological Survey and others have recently measured surface drainage return flows at selected areas in the valley. These data indicate an average salinity of about 4,000 mg/ℓ. Thus, as Duke et al. (1976) pointed out, if all return flows were through the drainage channels and phreatophyte consumptive use was not considered, the calculation of salt pickup would result in an estimated valley-wide contribution of approximately 660,000 tons. Consequently, the two salt loading figures, as predicted by inflow-outflow mass balancing and calculation using local data are sufficiently close to be confident in the values. Based on the figures pointed out in these preceding paragraphs, the salt loading due to irrigation in the Grand Valley can be segregated as follows:

1.	Canal and Ditch Seepage	23%;
2.	Lateral Seepage	32%;
3.	On-farm Losses	45%.

DEVELOPMENT OF FIRST LEVEL COST-EFFECTIVENESS FUNCTIONS

The array of salinity control alternatives applicable as first level measures in the Grand Valley are considered in four primary classes: (1) on-farm structural and operational improvements; (2) lateral lining by slip-form concrete or plastic pipeline; (3) concrete canal linings; and (4) collection and desalination of subsurface and surface drainage return flows. This list of basic salinity control measures is not intended to be exhaustive although these include those most

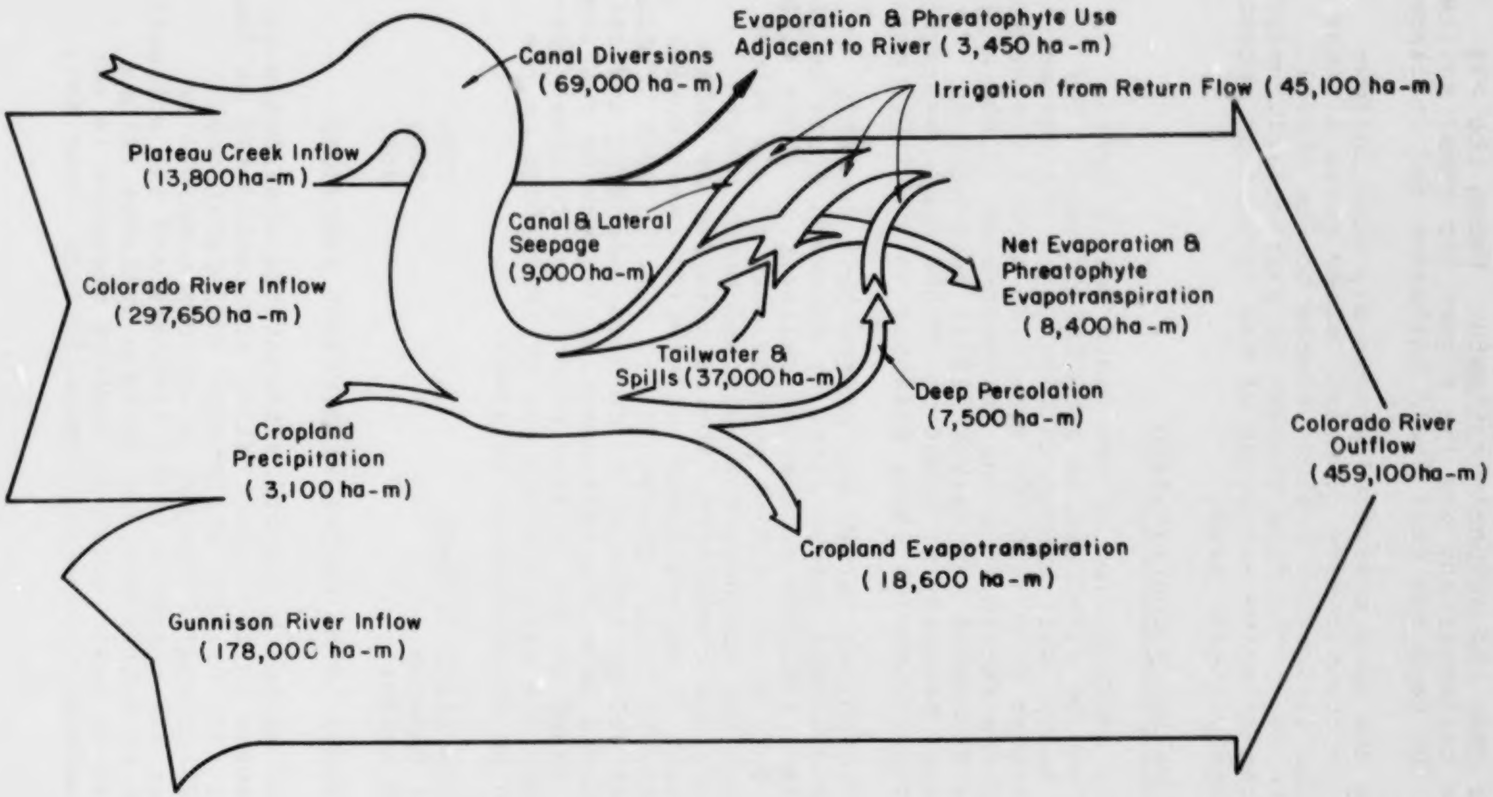

Evaporation & Phreatophyte Use
Adjacent to River (3,450 ha-m)
Canal Diversions
(69,000 ha-m)
Irrigation from Return Flow (45,100 ha-m)
Plateau Creek Inflow
(13,800 ha-m)
Canal & Lateral
Seepage
(9,000 ha-m)
Net Evaporation &
Phreatophyte
Evapotranspiration
(8,400 ha-m)
Colorado River Inflow
(297,650 ha-m)
Tailwater &
Spills (37,000 ha-m)
Deep Percolation
(7,500 ha-m)
Colorado River
Outflow
(459,100 ha-m)
Cropland
Precipitation
(3,100 ha-m)
Cropland Evapotranspiration
(18,600 ha-m)
Gunnison River Inflow
(178,000 ha-m)

Figure 20. Mean annual flow diagram of the Grand Valley hydrology.

likely to be actually authorized by the state and federal agencies responsible for controlling water quality.

At the first level there is one point that needs discussion. In the irrigation system, costs occur as either capital investments or annual operation-maintenance expenses. It is the feeling of the writer that O&M costs should not be factors in the evaluation of salinity control cost-effectiveness because the primary objective is to upgrade the existing system. Thus, the objective is to help an irrigator or conveyance company make more efficient use of water and thereby reduce the irrigation return flow volume, but not to directly subsidize farm production. To do so would be to violate the selection of "minimum cost" optimization criterion described in Section 4, even though the capital improvements themselves create production increases from the farm when better water management implies higher yields. The increases in yield would be very small in comparison to the "yields" realized by operation and maintenance support as part of a salinity control program. In an "about face", operation and maintenance costs are included in desalting facilities because they assume no broader purpose than salinity control in this analysis. If product water was sold to municipal, industrial, or agricultural users rather than returned to receiving waters such costs may not be included.

On-Farm Improvements

On-farm water management improvements which improve irrigation efficiency and thereby reduce return flows include: (1) improved irrigation practices implemented through irrigation scheduling; (2) structural rehabilitation; (3) conversion to more efficient methods of irrigation; and (4) relief or interceptor drainage.

Irrigation Scheduling --

Recent studies in Grand Valley have indicated that irrigation scheduling services, even when accompanied by flow measurement structures, generally do not significantly improve farm and application efficiencies (Skogerboe, et al. (1974a). A west-wide review of irrigation scheduling by Jensen (1975) indicated that a 10% improvement (from 40 to 50%) is realistically possible without system conversions or more energy intensive operations. In the Grand Valley, an irrigation scheduling service which included water measurement and farmer training would cost an estimated $30/ha and would reduce return flow salinity by about 20,000 metric tons annually. Since it is not known how irrigation efficiencies may be distributed, it is assumed that these figures may be linearly extrapolated yielding a cost-effectiveness function for irrigation scheduling of $37.50/ton with a limit of 20,000 metric tons amenable to this approach.

The overall impact of irrigation scheduling being only 10% of the total estimated on-farm potential improvement is insignificant by itself when considering the sensitivity of these type of costing estimates. Consequently, irrigation scheduling should be considered part of other measures rather than considered a separate alternative salinity control measure.

Structural Rehabilitation --

Irrigation efficiency can often be substantially improved by rebuilding and remodeling existing systems. The most commonly employed irrigation method in the valley is the furrow irrigation method. Structural improvements in this system may include concrete lined head ditches or gated pipe to reduce seepage losses, land leveling for better water application uniformity, adjusting field lengths and water application rates to be more congruent with soil and cropping conditions, and automation to provide better control. Flow measurement and scheduling services should accompany these types of improvements in order to maximize their effectiveness.

In the Grand Valley, head ditch requirements are generally less than the capacity of the smallest standard ditch available through local contractors (12 inch, 1:1 side slope, slip-form concrete). Consequently, lining costs can be expected to be linearly distributed. In Section 6, Eq. 73 was presented to estimate concrete lining costs (for small ditches the second term can be dropped):

$$C_c = 40.1Q^{0.56} \qquad \text{.......................................} \quad (147)$$

where,

C_c = total lining cost, \$/m; and
Q = channel capacity, m^3/sec.

Assuming an average head ditch capacity of 0.05 m^3/sec, Eq. 147 yields an estimated unit cost of \$7.50/m. This figure is well within the range encountered in the last two seasons in the valley. As noted earlier, six-inch diameter aluminum pipe costs approximately the same and can be arbitrarily substituted with equal cost-effectiveness. There are approximately 1.3 million meters of head ditches in the Grand Valley contributing an estimated 95,000 metric tons of salt to the river annually. If linings were assumed to be 90% effective, the cost-effectiveness of head ditch improvements would be \$113.40/ton (1.3 million meters x \$7.50/m ÷ 86,000 tons).

Automatic cutback furrow irrigation systems have demonstrated which, when combined with irrigation scheduling, may improve application efficiencies to 75 or 80%, thereby affecting an additional 60,000 ton decrease beyond the effects of the linings (Evans, 1977). In 1975, the installed cost of the cutback systems was $11.50/m. Thus, the salt load reductions by lining head ditch (86,000 tons) and the additional 60,000 m ton reduction increased application efficiency results in cost-effectiveness is $102.40/ton. Because of the small nature of these ditches, linear distribution can be assumed without introducing significant error. In the case of the Grand Valley, it appears automation may be added to surface irrigation systems for the additional efficiency at about the same cost-effectiveness as the simple head ditch or gated-pipe improvements. Where head ditch capacities are large, concrete lining would generally be more cost-effective than piped systems. Whether or not automated cutback would enjoy any advantage over regular linings under these conditions would require further evaluation.

Field lengths may be modified along with land shaping to improve the uniformity of water applications. This would be particularly true in soils having a relatively high infiltration capacity, but not as effective in tight soils such as those in the valley. There appear to be a few studies now underway which will yield good estimates of surface irrigation uniformity data on a field scale. However, at the time of this writing, there does not exist a satisfactory method of evaluating surface irrigation uniformities under variably sloped fields. Consequently, an analysis of the land shaping, run length alternatives has not been made for this study.

System Conversion --

In order to completely control irrigation return flows, the method of applying irrigation water needs to be independent of soil properties (sprinkler and trickle irrigation systems). In earlier sections, the application of sprinkler irrigation systems was shown to be approximately 80% efficient (application efficiency) whereas trickle systems could be expected to operate at the 90% level. Applying either system to the average field size in the Grand Valley (2-3 hectares) would be very expensive, so most systems would irrigate multiple fields. Figure 15 indicates that portable sprinkler irrigation systems (sideroll and handmove) would cost about $900 per hectare for coverages larger than 10 hectares. Trickle irrigation systems would cost approximately $1,800 per hectare for sizes greater than 2 hectares. Assuming irrigators would consolidate fields sufficiently to avoid the high cost applications on small fields, and assuming application efficiencies of 80% and 90% for sprinkler and trickle systems, respectively, the salt loading reduction for each system can be calculated as follows (an existing application efficiency of 64% is determined from the valley hydro-salinity data):

$$SLR = SCDS + \frac{TOFS}{TA}\left[1 - \frac{1-AE}{0.36}\right] \quad \text{..............................(148)}$$

where,

SLR = tons of salt loading reduced per hectare;
SCDS = tons per hectare reduced assuming the pressurized systems eliminate head ditches;
TOFS = total on-farm salinity, 190,900 tons;
TA = total irrigated acreage, 25,000 ha; and
AE = application efficiency expressed as a fraction.

Thus, for sprinkler systems the per hectare salt decrease is 6.84 tons and for trickle irrigation systems, 8.96 tons. Mobile or portable sprinkler systems would have average salinity cost-effectiveness ratios of approximately \$131.58/ton where the respective average for trickle systems would be about \$200.89/ton. Solid-set sprinklers would be at least double these figures and are therefore not evaluated. Center-pivot systems would be difficult to apply in the Grand Valley because of the small average size of land holdings.

Field Drainage --

The low permeability of Grand Valley soils dictate relatively close drain spacings (12-24 meters). Although field drainage has been proven partially effective in reducing salt pickup (Skogerboe, et al. 1974b), the costs are so high that drainage would not be competitive with other salinity control measures. Evans, et al. (1978a) report drainage cost-effectiveness values ranging in the thousands of dollars per ton. As a result, field drainage would not be included in any local salinity control policy.

Optimal On-Farm Improvement Strategies --

The first level cost-effectiveness function representing the on-farm salinity control alternative is developed by computing the minimum cost strategy at various levels of on-farm control. These results for the Grand Valley case are shown in Figure 21.

The actually computed cost-effectiveness relationship for on-farm improvements is the step function shown as the solid lines. This characteristic occurs because of the linear assumption regarding the distribution of costs and salinity impacts. The broken curve represents a best fit through the various discrete points and in itself actually creates the cost-effectiveness distributions avoided before. Field

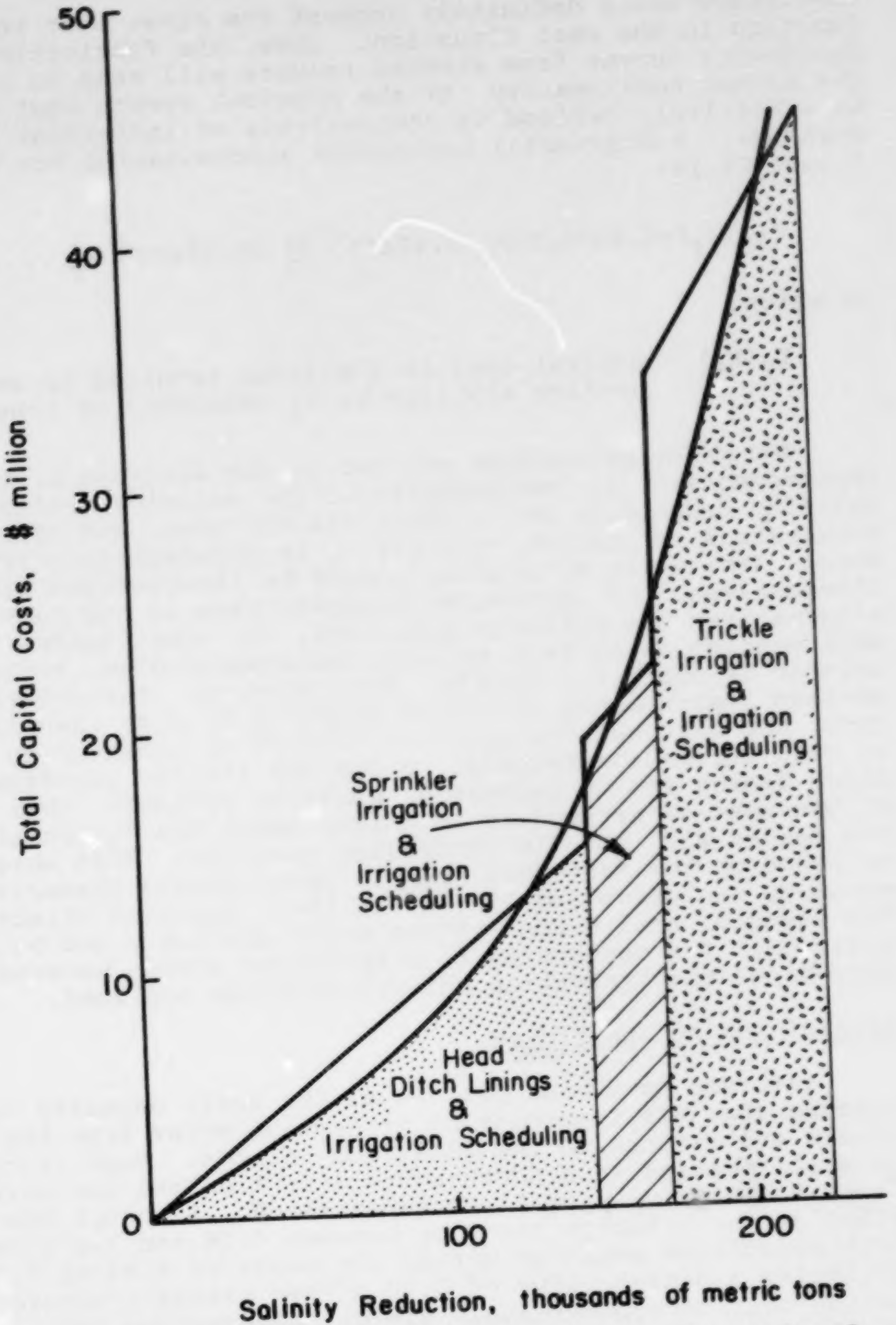


Figure 21. Cost-effectiveness function for the first level, on-farm improvement alternatives, in the Grand Valley.

experience would definitely support the curve over the step function in the real situation. Thus, the fabrication of continuous curves from stepped results will tend to re-introduce the actual nonlinearity of the physical system that could not be effectively defined in the analysis of individual on-farm measures. A polynomial regression approximating the curve in Figure 21 is:

$$Y_f\ (X_f)=0.03+0.10X_f-6.82\text{x}10^{-4}X_f^2+5.67\text{x}10^{-6}X_f^3 \quad \text{.........(149)}$$

in which,

$Y_f(X_f)$ = capital cost in \$ million required to reduce on-farm salinity by X_f thousands of tons.

Two major strategies evolved in the analysis of on-farm improvements: (1) improvements to the existing system creating salinity reductions up to about 150,000 tons; and (2) system conversions to provide controls up to approximately 220,000 tons. Irrigation scheduling should be incorporated with all alternatives. Of particular interest here is the fact that the alternatives are mutually exclusive. In other words, in implementing an on-farm salinity management plan, either one or another is optimally chosen. For instance, if planners selected on-farm improvements to reduce salinity by more than 150,000 tons, the alternatives would be limited to changing to sprinkler or drip irrigation methods. Below the 150,000 ton figure, head ditch lining and/or automation would be optimal. This structure of the cost-effectiveness is unique among the alternatives as the reader will note in succeeding sections. This uniqueness is based on the fact that on-farm improvements themselves are mutually exclusive and limited in their expected effectiveness. For example, head ditch linings would obviously not be considered in the conversion to a sprinkler system because this element of the irrigation network would be replaced.

Lateral Lining and Piping

Laterals have been defined as the small capacity conveyance channels transmitting irrigation water from the supply canals and ditches to the individual fields. Most of these laterals operate in a north-south direction and can carry the flows in relatively small cross-sections. Although the capacities of the laterals may vary between 0.06 and 1.4 m^3/sec, most capacities would be within the range of 0.06 to 0.20 m^3/sec. Utilizing a median value of 0.20 m^3/sec yields a concrete lining cost of approximately \$16/m. Alternative use of PVC pipe approximates concrete lining costs for this capacity and a further distinction will not be made. However, by this assumption the small seepage losses which would still occur from concrete lined channels are neglected.

As noted earlier, Grand Valley laterals extend approximately 600,000 meters, less than one half the length of field head ditches. Seepage under existing conditions contributes about 202,000 metric tons, or slightly less than the on-farm contribution. Although no attempt is made to distribute the lateral lining costs to account for variable capacity, the cost-effectiveness function for Grand Valley lateral lining is about \$49.50 per metric ton. Thus, the estimated costs of lining the total lateral system in the valley is about \$10 million.

Canal and Ditch Lining

There are fourteen major canal and ditch systems in the Grand Valley ranging in length from 74 kilometers for the Government Highline Canal (17 m^3/sec capacity) to 4 kilometers for the Mesa County Ditch (1 m^3/sec capacity). The pertinent parameters for each canal, along with the seepage contribution to salt loading, were substituted into Eq. 58. The resulting functions were then minimized using the Jacobian Differential Algorithm described in Section 4 for a range of salinity reductions accomplished from a canal lining program. These results are given in Figure 22 which shows the total capital construction costs as a function of the annual salt load reduction to be realized. The upper curve is the minimum cost associated with each value on the abscissa. Underneath the upper curve are the costs attributed to the various valley-wide canals. For example, if the contribution of canal seepage to the salt loading problem was to be reduced by 87,500 tons annually through linings, the capital construction cost would be approximately \$27 million with \$13.5 million on the Government Highline canal, \$8.7 million on the Grand Valley system, \$2.2 million on the small ditches (e.g., Price, Stub, etc.), \$1.6 million on the Orchard Mesa System, and the remainder on the Redlands System.

A regression equation for the canal lining cost-effectiveness function is:

$$Y_c(X_c) = -0.01+0.18X_c +4.92x10^{-4} X_c^2 +1.03x10^{-5} X_c^3 \; ....(150)$$

in which,

$Y_c(X_c)$ = canal lining cost, \$ million, to reduce salt loading by X_c thousands of tons.

The results obtained in optimizing canal lining policies are interesting in the sense that they demonstrate the need

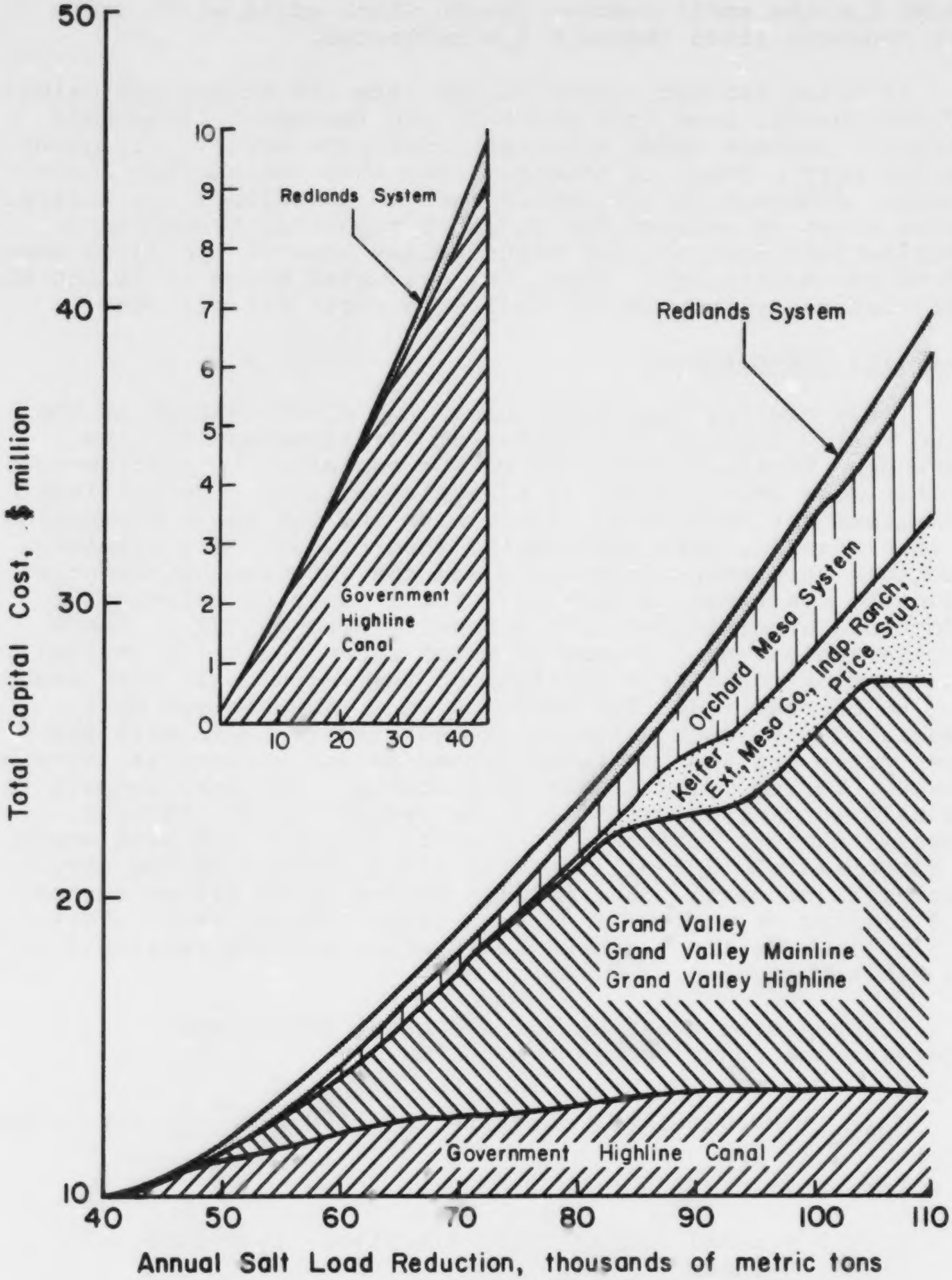


Figure 22. Optimal Grand Valley canal lining cost-effectiveness function.

to initiate linings on more than one segment of the conveyance system when full scale implementation begins. This may not be practical from a planning or scheduling stand point.

Desalination --

Desalting evaluations involved first determining the most cost-effective process and, second, the most cost-effective feedwater and brine disposal facitlities. The base condition used in Section 5 to compare desalting systems is a reasonable approximation of the Grand Valley situation. Consequently, the optimal desalting policy determined uitilizes a reverse osmosis system with feedwater wells and brine injection wells. To express desalting cost-effectiveness in the same format as the agricultural alternatives, the costs are plotted against the mass of salts removed from the system. For the purposes of this report, an interest rate of 7% and a usable life of 30 years will be assumed. For the reverse osmosis system, Figure 23 shows the resulting cost-effectiveness function.

It might be noted that whereas agricultural salinity control costs exhibit increasing marginal costs with scale, the opposite is true for desalting systems. In an optimizational analysis, therefore, the respective feasibility of desalting technology is maximized for large scale applications. For small systems, desalting is much less cost-effective than treatment of the agricultural system. As these factors are considered, a linear approximation representing the average marginal cost would serve at least as well as the non-linear function (note that the existing curve violates the convexity requirements of the Jacobian Differential Algorigthm). Consequently, desalting cost-effectiveness in the Grand Valley can be represented by:

$$Y_d(X_d) = 0.320\ X_d \qquad (151)$$

where,

$Y_d(X_d)$ = capitalized costs, \$ million, needed to remove X_d thousands of tons from the irrigation return flows.

DEVELOPMENT OF SECOND LEVEL COST-EFFECTIVENESS FUNCTIONS

The individual cost-effectiveness functions at the first level (desalting, canal lining, lateral lining, and on-farm improvements) are optimally integrated to determine the minimum cost salinity control strategy for the Grand Valley (level 2).

For purposes that will be discussed later, the individual cost-effectiveness functions for the level 1 alternatives might be transformed into dimensionless curves by dividing each

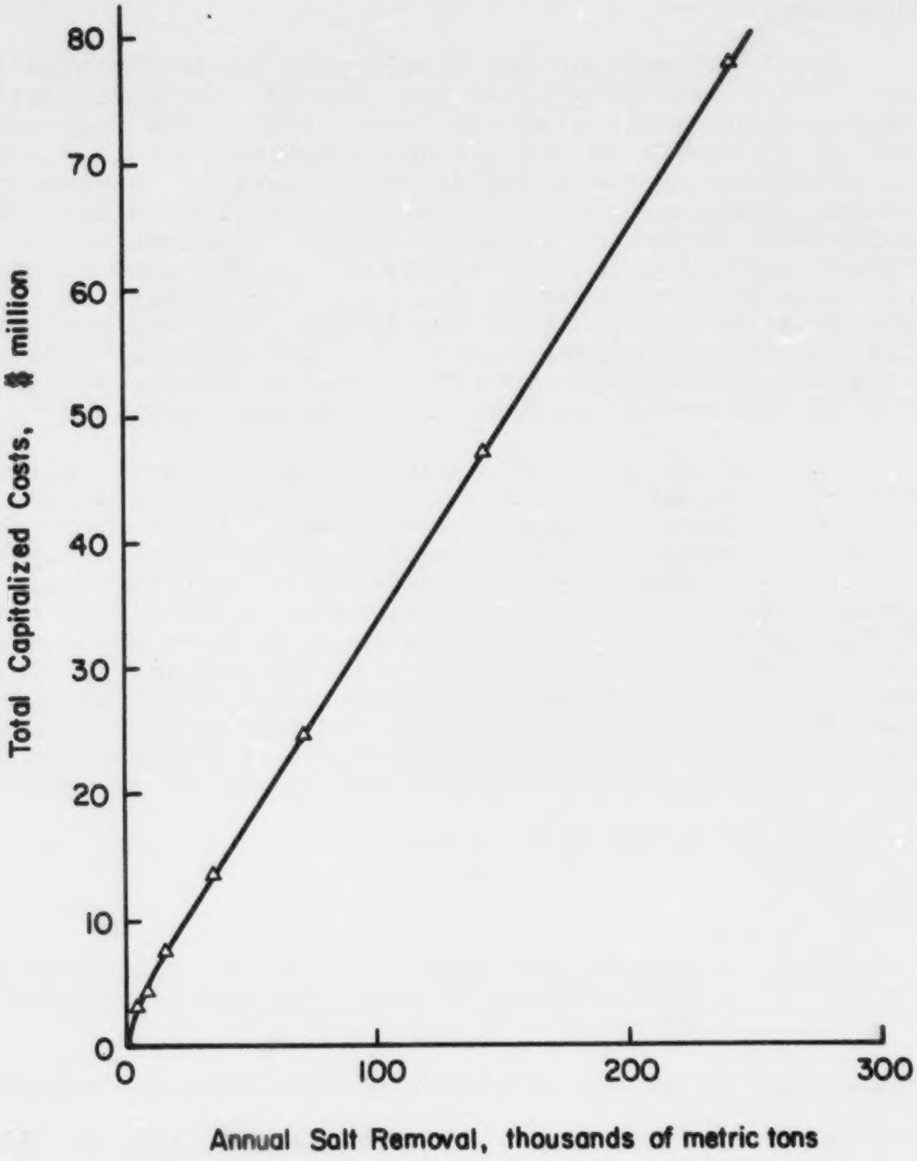


Figure 23. Grand Valley desalination cost-effectivenss function.

ordinate and abscissa point by the upper limit cost and salt loading reduction, respectively. A plot of the results is shown in Figure 24. The ordinate in Figure 24 is the fraction of the maximum costs expended for each level 1 alternative corresponding to an abscissa value of the fraction of the maximum salt load reduction. A polynomial regression fit through the various points gives the dimensionless relationships similar to Eqs. 149 and 150. The original cost-effectiveness functions can now be rewritten in the dimensionless form. For on-farm improvements:

$$Y_f(X_f)=49.23\left[0.0006+0.4418\left(\frac{X_f}{220}\right)-0.67\left(\frac{X_f}{220}\right)^2+1.228\left(\frac{X_f}{220}\right)^3\right] \quad \ldots\ldots (152)$$

Similarly for lateral linings, canal linings, and desalting:

$$Y_\ell\ (X_\ell) = 0.0495\ X_\ell \quad \ldots\ldots (153)$$

$$Y_c(X_c)=40.0\left[-2.5\text{x}10^{-4}+0.50\left(\frac{X_c}{110}\right)+0.16\left(\frac{X_c}{110}\right)^2+0.34\left(\frac{X_c}{110}\right)^3\right] \quad \ldots\ldots (154)$$

$$Y_d(X_d) = 0.320\ X_d \quad \ldots\ldots (155)$$

where,

$Y_i(X_i)$ = capital cost, \$ million, required to diminish return flow salinity X_i thousands of tons.

The cost-effectiveness function at the second level, corresponding to the best management practices in the Grand Valley, is determined by solving the following optimization problem for the expected range of X_T:

$$Y_2 = \min\left[Y_f(X_f)+Y_\ell(X_\ell)+Y_c(X_c)+Y_d(X_d)\right] \quad \ldots\ldots (156)$$

subject to,

$$X_f + X_\ell + X_c + X_d = X_T \quad \ldots\ldots (157)$$

$$220{,}000 - X_f \geq 0 \quad \ldots\ldots (158)$$

$$202{,}000 - X_\ell \geq 0 \quad \ldots\ldots (159)$$

$$110{,}000 - X_c \geq 0 \quad \ldots\ldots (160)$$

$$630{,}000 - X_d \geq 0 \quad \ldots\ldots (161)$$

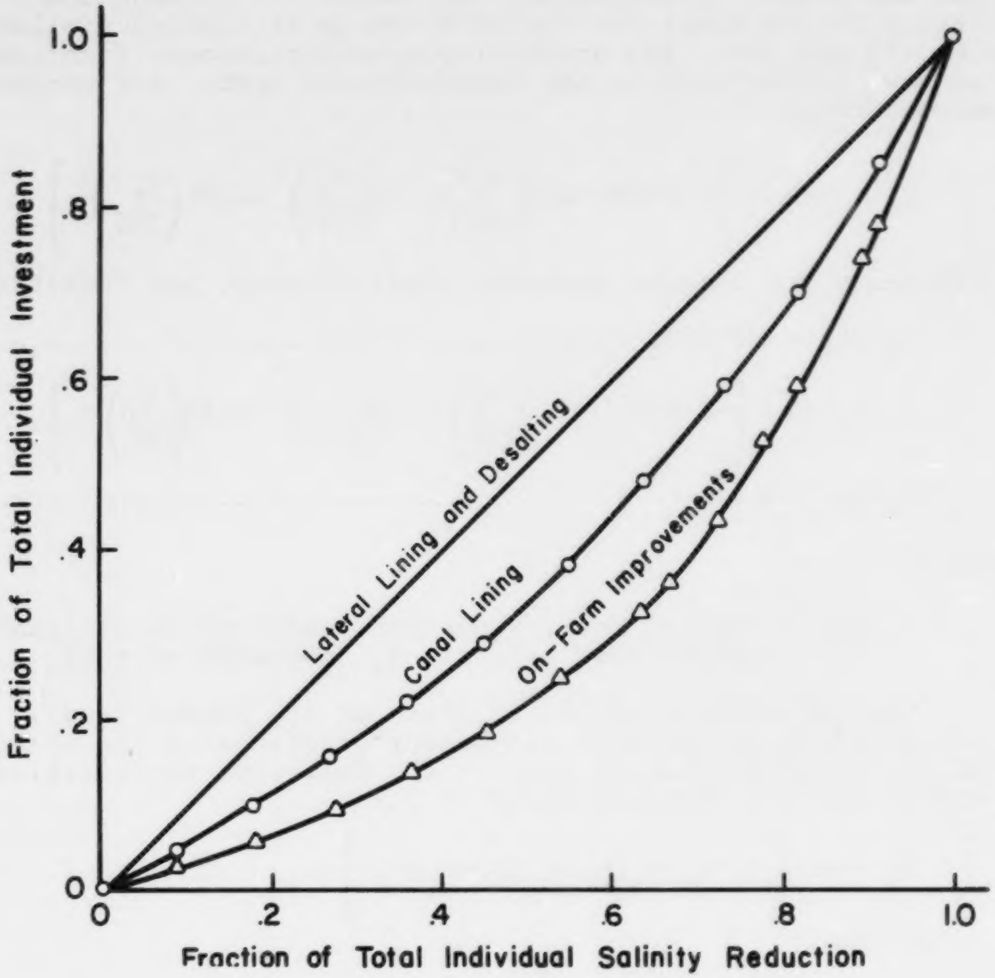


Figure 24. Dimensionless level 1 cost-effectiveness curves for the Grand Valley.

These expresions can be placed in appropriate use within the optimization procedure described in Section 4 or any other suitable techniques. The results shown in Figure 25 are approximated within about 13% by the equation:

$$Y_2 = -0.34+.0325X_T+1.87x10^{-5}X_T^2 +3.72x10^{-7}X_T^3 \text{ ....} \quad (162)$$

DISCUSSION OF RESULTS

In Section 4 the philosophy behind the multilevel optimization approach to evaluating salinity control strategies was discussed from both a theoretical and general viewpoint. The results presented in this section might now be examined to illustrate earlier explanations relative to interpreting the results of the analysis.

Consider three points on the Grand Valley cost-effectiveness curve (Figure 25): (1) total costs = $15 million, salt loading reduction = 266,000 tons; (2) total costs = $40 million, salt loading reduction = 403,000 tons; and (3) total costs = $80 million, salt loading reduction = 530,000 tons. For convenience, these three points have been designated as Cases 1, 2, and 3, respectively.

If the expenditure in the Grand Valley is to be $15 million in 1976 value dollars (Case 1), the optimal strategy in so doing is found from a vertical trace at this point on the curve representing the valley (Figure 25). Specifically, $10 million should be invested in lateral linings and $5 million in on-farm improvements. Referring back to the paragraphs on lateral lining, it is noted that a $10 million investment covers the cost of lining the entire system. Thus, for Case 1, the first part of the strategy is to line the lateral system entirely. In a similar backward look to Figure 21 representing the level 1 relationship for on-farm improvements, it is seen that a $5 million dollar cost corresponds to about a 64,000 ton reduction in the on-farm salinity contribution, and is so accomplished by head ditch linings, or cutback irrigation, and irrigation scheduling.

A $40 million dollar salinity control investment (Case 2) in the Grand Valley is seen from Figure 25 to reduce salinity by 403,000 metric tons by spending $10 million lining the lateral system, $20 million making on-farm improvements and $10 million lining some of the major canals. Referring again to Figure 21, a $20 million investment in on-farm improvements implies reducing the on-farm salt contribution by 156,000 tons by irrigating nearly all of the irrigated land with portable or mobile sprinkler systems. Figure 22 shows that $10 million dollars in canal lining would accomplish a 45,000 ton reduction

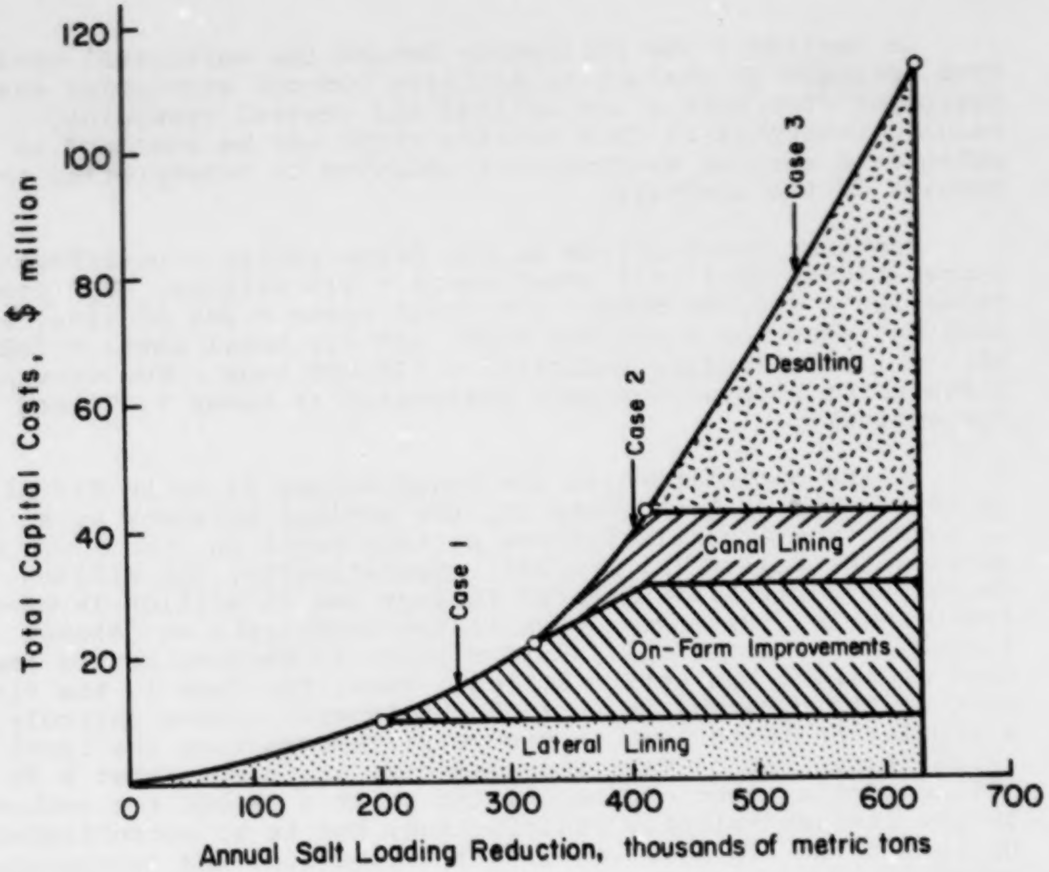


Figure 25. Grand Valley second level salinity control cost-effectiveness function.

in salt loading. To do this, a small (essentially insignificant) amount of lining should occur in the Redlands system with the remainder being applied to the Government Highline Canal.

Case 3 shows that agricultural improvements should stop at $42.6 million with any remaining salt volumes to be removed from the system through desalination of the subsurface irrigation return flows. The $42.6 million figure for local agriculture includes $10 million for lateral lining, $21 million for on-farm improvements, and $11.6 million in canal lining. On-farm improvements would still involve conversion to sprinkler irrigation. Canal lining strategies involve somewhat enlarged versions of the Case 2 results.

The mathematics of this analysis indicates the minimum cost salinity control strategy in the Grand Valley, but a planner or administrator must also consider the practicality of the solutions. For example, in the third case, it would probably be unrealistic to line a very small portion of the Redlands Canal system and a decision would be made to invest all of the funds into lining the required length of the Government Highline Canal. Likewise, on-farm improvements may be limited to converting the existing system to a sprinkler irrigated system with some measure of control to increase application efficiencies beyond those assumed for this analysis. The point to be made is that at this level of investigation, the inherent assumptions allow a certain amount of flexibility to account for some of the intangible social-institutional factors involved in an implementation effort.

In representing what might be called the best management practices for the Grand Valley, it must be realized that the four major implementation alternatives (lateral lining, on-farm improvements, canal linings, and desalting) only represent "structural measures." Consequently, nonstructural alternatives such as land retirement, influent and effluent standards, taxation, and miscellaneous enforcement options are not included. Nevertheless, the value of the sort of analysis can be clearly demonstrated. In the Grand Valley a plan might be proposed in which all of the canals would be lined, all of the laterals lined, and some on-farm improvements to reduce salt loading by 450,000 metric tons annually. Looking at Figures 21 and 22 and the comments in the paragraph describing lateral linings shows a total cost of such a program of $65.5 million. Figure 25 indicates the same reduction could be accomplished with a $54 million investment if the on-farm role were expanded, the canal lining program diminished and a limited desalting capacity were included. Thus, this optimization analysis illustrates how a $11.5 million savings (21%) can be achieved.

The eventual program in the Grand Valley is dictated by its respective feasibility in comparison to similar cost-effectiveness studies on the other subbasins in the Upper Colorado River Basin. In fact, the level of investment in the entire river system for salinity control depends on the level of damages created by the salinity. Since the completed four level analysis is not available, it is interesting to compare downstream damage with costs in the Grand Valley. Note that the estimates of marginal cost and downstream detriments must be the same. Walker (1975) reviewed much of the literature descriptive of the California, Arizona, and Republic of Mexico damages. At the time, Valentine (1974) had proposed damages of $175,000 per mg/ℓ of increase at Hoover dam ($146 per ton in Grand Valley assuming 8% interest). Other estimates in terms of equivalent damages attributable to Grand Valley range upward. A representative figure is $190/ton as proposed by the Bureau of Reclamation (Leathers and Young, 1976). Some as yet unpublished figures now place these damage figures as high as $375/ton. If the minimum cost curve in Figure 25 is differentiated to approximate marginal costs and be congruent with these damage figures, the $146 per ton damage estimate of Valentine (1974) falls at a 300,000 ton reduction, while the $190 per ton and $375/ton figures occur at 355,000 tons and 538,000 tons, respectively. Figure 26 is a plot of the marginal Grand Valley salinity costs as a function of salt loading reductions. Thus, not considering secondary benefits in the Grand Valley, or obviously all the consequences in the lower basin, the level of investment in the Grand Valley could range between $19 million and $83 million. In any event, it can be seen that the actual policy for salinity control in a subbasin depends on decisions made at higher levels. Similarly, within a subbasin the measures implemented to control salinity change as the emphasis on the subbasin itself changes.

EVALUATION AT THE THIRD AND FOURTH LEVELS

In demonstrating this approach to planning salinity control strategies on a large scale, the Grand Valley was used as a case study because of its data base. Since no other area in the Upper Colorado is well enough defined to allow similar developments, it may appear impossible to carry the analysis to its conclusions at the fourth level. At the same time, without the fourth level of analysis, the optimal program in the irrigated areas like Grand Valley cannot be effectively established.

It is not a difficult task to estimate the total costs necessary for lining all canals, ditches, and laterals in an irrigated area if some information is generally available. Likewise, the array of on-farm improvements may also be described in terms of an estimated total cost. Examinations of stream gaging records will give a reasonable estimate of the salinity

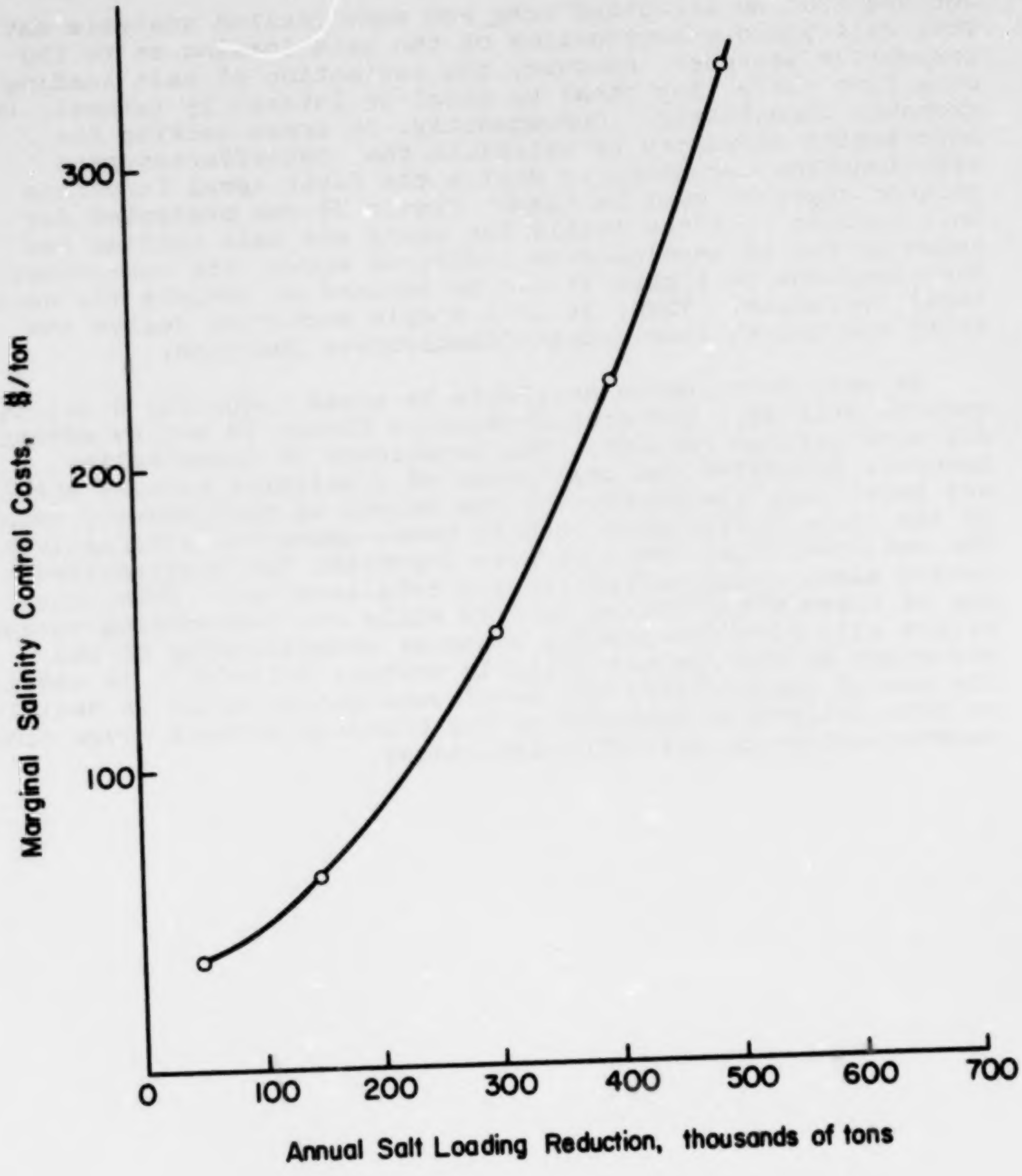


Figure 26. Marginal cost function of optimal salinity control strategy in the Grand Valley.

derived from an irrigated area and some limited analysis may very well yield a segregation of the salt loading as to the respective sources. However, the estimation of salt loading on a fine scale, say canal by canal or lateral by lateral, is probably unrealistic. Consequently, in areas lacking the information necessary to establish the cost-effectiveness relationships necessary to derive the first level functions another approach must be taken Figure 24 was presented for this purpose. If the totals for costs and salt loading are known or can be developed as indicated above, the non-linear distributions in Figure 24 can be assumed to compute the second level functions. Then, it is a simple matter to derive the third and fourth level cost-effectiveness function.

As more data become available in areas requiring a salinity control strategy, the distribution in Figure 24 can be adjusted for more refined results. The experience in Grand Valley, however, indicates the components of a salinity control strategy are relatively insensitive to the degree of nonlinearity because of the large differences in unit costs among the alternatives. The nonlinearities would be more important for alternatives having similar cost-effectiveness relationships. Thus, the use of these Grand Valley results while not introducing serious errors will give the planner a better understanding of the structure of the optimal salinity control policies. In addition, the use of these curves may be of substantial value in deciding on data collection programs as the planning process moves from reconnaissance to definite plan stages.

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APPENDIX A

DESALTING COST ANALYSIS COMPUTER CODE

DESCRIPTION OF CODE

In an earlier section, the costs of various desalting systems were described. A set of cost estimating procedures published by the U. S. Department of Interior (1972) were mathematically simulated and coded in Fortran IV.

The desalting model listed in the following pages consists of a main program, DESALT1, and five subroutines with call statement data transfer, DESCONT A, DESALTC A, ADJUST A, OUTPUT1 A, and WRITE A. The composite model requires about 34,000 bytes of central memeory storage and executes in 3-5 control processor seconds per analysis.

The main program DESALT1 serves only as an data input device. Control variables are entered to manage several input and output data destiny options as described within the listing itself. The definitions of each input variable are also given in the listing. Input data may be printed with subroutine WRITE as illustrated in Table A-1. After control and input data are read in, Subroutine DESCONT is called to coordinate the primary desalting cost analysis. Subroutine DESCONT first calls DESALTC which computes the capital and operation and maintenance costs for whatever process is specified. It then calls ADJUST to determine feedwater and brine disposal costs. And finally, DESCONT directs the information to OUTPUT1 for output. The output can be plant costs, feedwater-brine costs, or total costs. An example of the model output for total costs is shown in Table A-2.

TABLE A-1. INPUT DATA PRINTOUT FROM EXAMPLE ANALYSIS

```
     INPUT DATA

**************************************************************************

PLANT VARIABLES                  FEEDWATER AND BRINE VARIABLES
 1. CAPACITY=      MGD            1. PRIME MOVER COST INDEX =   2.13
 2. USE FACTOR=  .90              2. CONCRETE PIPE COST INDEX =   1.00
 3. ENR BLDG INDEX=1354.0         3. PIPELINE PUMPING PLANT COST INDEX =    1.90
 4. BLS LABOR INDEX= 4.93         4. EARTHWORK COST INDEX =   1.92
 5. BLS CHEM INDEX= 181.0         5. SOURCE OF FEEDWATER- WELLS
 6. FIX CHG RATE=.08560           6. AVERAGE WELL DEPTH =   328. FT
 7. INT RATE=  .07                7. AVERAGE DISTANCE FROM PLANT =  3280. FT
                                  8. TYPE OF BRINE DISPOSAL- EVAPORATION PONDS
WATER CHARACTERISTICS             9. ANNUAL EVAPORATION RATE =     4. FT
 1. FEED TEMP (DEG F)=60.00
 2. FEED TDS (PPM)= 4000.0
 3. FEED NA (PPM)=1260.
 4. FEED K (PPM)=    8.
 5. FEED HCO3 (PPM)= 106.
 6. FEED NO3 (PPM)=    0.
 7. FEED SO4 (PPM)=1075.
 8. FEED CL (PPM)=2035.
 9. FEED CA (PPM)= 393.
10. PRODUCT TDS (PPM)= 500.

MISCELLANEOUS
 1. ELEC RATE= 20.00000
       ($/1000 KWH)
 2. FUEL RATE=  1.14000
       ($/MBTU)
 3. LAND PRICE=  .002000
    ($MILLION/ACRE)
```

TABLE A-2. EXAMPLE COST ANALYSIS FOR A REVERSE OSMOSIS DESALTING SYSTEM SUPPLIED BY FEEDWATER WELLS AND DISPOSING OF BRINE THROUGH INJECTION WELLS

RO DESALTING PROCESS COST ANALYSIS

COST DESCRIPTION	DESALTING PLANT CAPACITIES							
	.25 MGD	.50 MGD	1.0 MGD	2.0 MGD	4.0 MGD	8.0 MGD	16.0 MGD	32.0 MGD
CAPITAL COSTS/10**6								
A. CONSTRUCTION	1.73	2.43	3.80	6.50	11.81	22.47	43.89	86.29
B. STEAM FACILITIES	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
C. SITE DEVELOPMENT	.06	.09	.14	.22	.35	.55	.87	1.37
D. INTEREST-(CONSTR)	.04	.07	.12	.22	.43	.86	1.77	3.67
E. START-UP	.00	.01	.01	.02	.04	.08	.15	.30
F. GENERAL EXPENSE	.21	.29	.44	.71	1.20	2.12	3.84	7.02
G. LAND	.12	.23	.45	.90	1.80	3.59	7.17	14.36
H. WORKING CAPITAL	.01	.02	.03	.06	.11	.20	.39	.75
SUBTOTAL	2.18	3.13	4.99	8.63	15.73	29.87	58.09	113.74
ANNUAL COSTS/10**3								
I. LABOR-MATERIALS	42.5	57.7	80.4	121.4	193.1	318.0	538.4	925.6
J. CHEMICALS	7.1	14.2	28.5	57.0	113.9	227.8	455.6	911.2
K. FUEL	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
L. STEAM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
M. ELECTRICITY	21.1	42.1	84.0	167.5	334.3	666.8	1331.6	2659.1
N. REPLACEMENT	8.2	16.4	32.8	65.7	131.4	262.8	525.6	1051.2
SUBTOTAL	79.0	130.5	225.7	411.6	772.7	1475.5	2851.3	5547.1
TOTAL ANNUAL COSTS								
O. ANNUAL COSTS/10**3	263.	395.	645.	1135.	2090.	3973.	7786.	15048.
P. WATER COST-$/1000 G	2.88	2.16	1.77	1.56	1.43	1.36	1.32	1.29
Q. SALT COSTS-$/TON	86.53	64.84	53.04	46.65	42.93	40.82	39.58	38.64
SALT REMOVED (TONS)	3042.	6085.	12169.	24338.	48677.	97353.	194707.	389413.

SALT COSTS ($/TON) = 15.73140/(CAPACITY (MGD))** .8167951 + 37.71481

SUMMARY IN METRIC UNITS

	DESALTING PLANT CAPACITIES							
	.95	1.89	3.79	7.57	15.14	30.28	60.57	121.13
	THOUSANDS OF CUBIC METERS PER DAY							
TOTAL ANNUAL COSTS								
P. WATER COST-$/M**3	.76	.57	.47	.41	.38	.36	.35	.34
Q. SALT COSTS-$/MTON	95.38	71.47	58.46	51.42	47.32	44.99	43.63	42.60
SALT REMOVED (MTONS)	2760.	5520.	11040.	22080.	44159.	88319.	176638.	353276.

MAIN PROGRAM LISTING

```
      PROGRAM DESALT1
     1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
C
C*** DESCRIPTION OF PROGRAM VARIABLES ***
C             (ENGLISH UNITS)
C
C  1. DESALTING PLANT PARAMETERS-
C    A. NAME   TYPE OF PROCESS
C    B. UF     USE FACTOR (FRACTION OF TIME IN USE)
C    C. CP     PRODUCT WATER CAPACITY (MGD)
C    D. CW     COOLING WATER CAPACITY (MGD)
C    E. CB     BRINE CAPACITY (MGD)
C    F. K1     IDENTIFICATION CODE FOR PROCESSES
C           K1=1 - MSF
C           K1=2 - VTE-MSF
C           K1=3 - VC-VTE-MSF
C           K1=4 - ED
C           K1=5 - RO
C           K1=6 - VF-VC
C           K1=7 - IX
C    G. K      CAPACITY CODE
C            K=1 - CP=0.25 MGD
C            K=2 - CP=0.50 MGD
C            K=3 - CP=1.00 MGD
C            K=4 - CP=2.00 MGD
C            K=5 - CP=4.00 MGD
C            K=6 - CP=8.00 MGD
C            K=7 - CP=16.00 MGD
C            K=8 - CP=32.00 MGD
C
C  2. FEEDWATER PARAMETERS-
C    A. IFWC   SOURCE OF FEEDWATER
C           IFWC=1 - WELLS
C           IFWC=2 - SURFACE DIVERSION
C    B. TEMP   TEMPERATURE (DEG F)
C    C. TDSI    SALINITY CONCENTRATION OF FEEDWATER (MG/L)
C    D. TDSO    SALINITY CONCENTRATION OF PRODUCT (MG/L)
C    E. CAI,NAI,KI,MGI,HCO3I,SO4I,CLI,AND NO3I ARE THE RESPECTIVE IONIC
C        SPECIES CONCENTRATION (MG/L)
C    F. DEPTHF  DEPTH OF FEEDWATER WELLS (FEET)
C    G. DISTF   AVERAGE DISTANCE TO FEEDWATER SOURCES (FEET)
C
C  3. BRINE DISPOSAL PARAMETERS-
C    A. IBC     TYPE OF DISPOSAL
C          IBC=1 - INJECTION WELLS
C          IBC=2 - EVAPORATION PONDS
C    B. E       ANNUAL EVAPORATION RATE (FEET)
C    C. DEPTHB  AVERAGE DEPTH OF INJECTION WELLS (FEET)
C    D. DISTB   AVERAGE DISTANCE TO INJECTION WELLS (FEET)
C
C  4. ECONOMIC PARAMETERS-
C    A. ENRI    ENGR NEWS RECORD CONST COST INDEX
C    B. BLS1    BUR LABOR STAT LABOR COST INDEX (SIC 494-7)
C    C. BLS2    BUR LABOR STAT CHEM AND ALLIED PRODUCT COST INDEX
C    D. PMI     USBR PUMP AND PRIME MOVER COST INDEX
C    E. CPI     USBR CONCRETE PIPELINE COST INDEX
C    F. PPI     USBR PUMPING PLANT COST INDEX (PIPELINE)
C    G. EI      USBR CANAL AND EARTHWORK COST INDEX
C    H. FCR     FIXED CHARGE FACTOR FOR DEPRECIATING CAPITAL
```

```
C    I. IR      INTEREST RATE FOR NON-DEPRECIATING CAPITAL
C    J. EC      ELECTRICITY RATES ($/1000 KWH)
C    K. FR      FUEL COSTS ($/MBTU)
C    L. LP      LAND PRICE ($MILLION/ACRE)
C
C  5. MISCELLANEOUS CONTROL PARAMETERS-
C    A. IWRITE  INPUT DATA LISTING
C            IWRITE=1 - LISTING
C            IWRITE=2 - NO LISTING
C    B. IWRITE1 FORMAT OF PROGRAM OUTPUT
C          IWRITE1=1 - OUTPUT OF TOTAL COSTS
C          IWRITE1=2 - OUTPUT OF FEEDWATER SUPPLY AND BRINE DISPOSAL COSTS
C          IWRITE1=3 - OUTPUT OF PLANT COSTS
C
C                    *****
C
C
      REAL NAI,KI,NO3I,IR,LP
      DIMENSION NAME(7,10)
      READ(5,101) ((NAME(I,J),J=1,10),I=1,7)
      READ(5,100)DISTB, UF,ENRI,BLS1,BLS2,FCR,IR,TEMP
      READ(5,100) NAI,KI,HCO3I,NO3I,SO4I,TDSI,CAI,TDSO
      READ(5,100) DEPTHF,PMI,CPI,DISTF,PPI,E,EI,DEPTHB
      READ(5,100) EC,FR,LP,CLI
      READ(5,894) ICTC,IFWC,IBC,IWRITE,IWRITE1
      DO 12 I3=1,6
      K1=I3
      TDSI=1000.
      DO 12 I1=1,2
      TDSI=TDSI+2000.
            CALL DESCONT(NAME,DISTB,UF,ENRI,BLS1,BLS2,FCR,IR,TEMP,NAI,
     1KI,HCO3I,SO4I,TDSI,CAI,TDSO,DEPTHF,PMI,CPI,DISTF,PPI,E,EI,DEPTHB,
     2EC,FR,LP,CLI,IGTC,IFWC,IBC,IWRITE,IWRITE1,K1,CP,K)
   10 CONTINUE
   12 CONTINUE
  100 FORMAT(8F10.1)
  101 FORMAT(10A8)
 894  FORMAT(24I2)
C
C*** THIS PROGRAM CODE IS WRITTEN IN TERMS OF ENGLISH UNITS.  TO USE
C    METRIC DATA OR HAVE RESULTS IN METRIC UNITS, USE FOLLOWING
C    CONVERSIONS-
C
C        ENGLISH         TO              METRIC          MULTIPLY BY
C
C                                        M**0/DAY        3785.41
C        MGD                             METERS              .3048
C        FEET                            $MILLION/HA        2.47097
C        $MILLION/ACRE                   $/MJOULES       1055.
C        */MBTU                          $/MJOULES       3599.97
C        $/1000 KWH
C
C        DEG C = (DEG F -32.)/1.8
C
C        *****
C
     STOP
     END
```

CODE LISTING FOR SUBROUTINE DESCONT

```
      SUBROUTINE   DESCONT(NAME,DISTB,UF,ENRI,BLS1,BLS2,FCR,IR,TEMP,NAI,
     1KI,HCO3I,SO4I,TDSI,CAI,TDSO,DEPTHF,PMI,CPI,DISTF,PPI,E,EI,DEPTHB,
     2EC,FR,LP,CLI,ICTC,IFWC,IBC,IWRITE,IWRITE1,K1,CP,K)
      REAL NAI,KI,NO3I,IR,LP
      DIMENSION A1(8),A2(8),A3(8),A4(8),A5(8),A6(8),A7(8),A8(8),A9(8),
     1A10(8),A11(8),A12(8),A13(8),A14(8),A15(8),A16(8),A17(8),A18(8),
     2NAME(7,10)
      IF(IWRITE.EQ.1) CALL WRITE(CP,UF,ENRI,BLS1,BLS2,FCR,IR
     1,TEMP,TDSI,NAI,KI,HCO3I,NO3I,SO4I,CLI,CAI,TDSO,EC,FR,LP,PMI,CPI,EI
     1,DEPTHF,DISTF,DEPTHB,DISTB,IFWC,IBC,E,PPI)
      IWRITE=10
      TDSP=TDSO
      ICODE=K1
      CP=.125
      DO 1111 K=1,8
      CP=2.*CP
      CALL DESALTC(CP,UF,ENRI,BLS1,BLS2,FCR,IR,TEMP,TDSI,NAI,KI,HCO3I,
     1NO3I,SO4I,CAI,TDSP,EC,FR,LP,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
     2,C13,C14,NAME,GB,BPR,ICODE,CW,CLI,CT1,CT2,CT3,CT4,CT5,COW,COS,CBO,
     1ICTC,IFWC,DEPTHF,PMI,   CPI,DISTF,PPI,E,EI,IBC,DEPTHB,DISTB,TONS)
      CALL ADJUST(ICTC,ENRI,CW,IFWC,CP,DEPTHF,PMI,CPI,DISTF,PPI,E,EI,IBC
     1,DEPTHB,LP,EC,BLS1,C1A,C2A,C3A,C4A,C5A,C6A,C7A,C8A,C9A,C10A,C11A,
     2C12A,C13A,C14A,DISTB,CB,IR)
      IF(IWRITE1-3) 1,2,2
    1 IF(IWRITE1-2) 3,4,4
    3 C1=C1+C1A
      C2=C2+C2A
      C3=C3+C3A
      C4=C4+C4A
      C5=C5+C5A
      C6=C6+C6A
      C7=C7+C7A
      C8=C8+C8A
      C9=C9+C9A
      C10=C10+C10A
      C11=C11+C11A
      C12=C12+C12A
      C13=C13+C13A
      C14=C14+C14A
      CT5=C9+C10+C11+C12+C13+1000.*((C1+C2+C3+C4+C5+C6)*FCR+(C7+C8)*IR)+
     1C14
      COW=CT5/CP/365.
      COS=CT5*1000./GB/1.5234/CBO
      GO TO 2
    4 C1=C1A
      C2=C2A
      C3=C3A
      C4=C4A
      C5=C5A
      C6=C6A
      C7=C7A
      C8=C8A
      C9=C9A
      C10=C10A
      C11=C11A
      C12=C12A
      C13=C13A
      C14=C14A
      CT5=C9+C10+C11+C12+C13+1000.*((C1+C2+C3+C4+C5+C6)*FCR+(C7+C8)*IR)+
     1C14
      COW=CT5/CP/365.
      COS=CT5*1000./GB/1.5234/CBO
    2 CONTINUE
      A1(K)=C1
      A2(K)=C2
      A3(K)=C3
      A4(K)=C4
      A5(K)=C5
      A6(K)=C6
      A7(K)=C7
      A8(K)=C8
      A9(K)=C9
      A10(K)=C10
      A11(K)=C11
      A12(K)=C12
      A13(K)=C13
      A14(K)=C14
      A15(K)=CT5
      A16(K)=COW
      A17(K)=COS
      A18(K)=TONS
      IF(K.LE.7) GO TO 1111
           CALL OUTPUT1(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12, A13,A14
     1,A15,A16,A17,A18,NAME,K1)
 1111 CONTINUE
      RETURN
      END
```

PLANT CAPITAL AND OPERATION-MAINTENANCE COSTS

```
      SUBROUTINE DESALTC(CP,UF,ENRI,BLS1,BLS2,FCR,IR,TEMP,TDSI,NAI,KI,
     1HCO3I,NO3I,SO4I,CAI,TDSP,EC,FR,LP,C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,
     2C11,C12,C13,C14,NAME,CB,BPR,ICODE,CW,CLI,CT1,CT2,CT3,CT4,CT5,COW,
     1COS,CRO,ICTC,IFWC,DEPTHF,PMI,CPI,DISTF,PPI,E,EI,IBC,DEPTHB,DISTB,
     2TONS)
      REAL IR,NAI,KI,NO3I,LP
      DIMENSION NAME(7,10)
      IF(ICODE.EQ.1) GO TO 1
      IF(ICODE.EQ.2) GO TO 2
      IF(ICODE.EQ.3) GO TO 3
      IF(ICODE.EQ.4) GO TO 4
      IF(ICODE.EQ.5) GO TO 5
      IF(ICODE.EQ.6) GO TO 6
      IF(ICODE.EQ.7) GO TO 7
      IF(ICODE.LT.1) GO TO 1000
      IF(ICODE.GT.7) GO TO 1000
C
C        ***  COST ANALYSIS FOR MSF SYSTEMS  ***
C
    1 C1=ENRI/952.*1.48*CP**.90456
      IF(CP.LE.50.) C1=ENRI/952.*1.9*CP**.83817
      IF(CP.LE.6.) C1=ENRI/952.*2.3*CP**.73444
      C2=ENRI/952.*.23*CP**.8015
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C2+C3)*IR/24.*(15.5*CP**.2119)
      C6=.119*(C1+C2+C3)**.9
      C7= LP*(2.+.2168*CP+.377*CP**.7756)
      C9=47.*CP**.6657
      IF(CP.LE.40.) C9=60.*CP**.6014
      IF(CP.LE.8.) C9=81.*CP**.458
      IF(CP.LE.3.) C9=95.*CP**.3042
      IF(CP.LE.1.) C9=95.*CP**.18182
      C9=C9*BLS1/3.76
      C10= BLS2/104.4*7.3*UF*CP
      XP=.58*CP**1.006
      C11= FR*445.3*XP**.986
      C12=64.*XP**.5476
      IF(XP.LE.40.) C12=70.*XP**.5
      IF(XP.LE.6.) C12=78.*XP**.44898
      IF(XP.LE.1.5) C12=80.*XP**.3571
      C12=C12*BLS1/3.76
      C13=EC*3.65*CP**.949
      C14=0.
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      CBR1=(1.-50./TDSI)/(900./CAI-1.)
      CBR2=(TDSI-50.)/(60000.-TDSI)
      IF(CBR1.LE.CBR2) BPR=CBR2
      IF(CBR1.GT.CBR2) BPR=CBR1
      CB=BPR*CP
      CW=CP*(2.5-BPR)
      IF(ICTC.EQ.1) CW=CP*(4.2-BPR)
      TDSP=50.
      GO TO 1000
C
C        ***  COST ANALYSIS FOR VTE-MSF SYSTEMS  ***
C
    2 C1=1.5964*CP**.8287*ENRI/952.
      IF(CP.LE.25.) C1=ENRI/952.*2.03965*CP**.74537
      IF(CP.LE.9) C1=ENRI/952.*2.57279*CP**.64352
      C2=ENRI/952.*.23*CP**.8015
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C2+C3)*IR/24.*14.*CP**.2389
      C6=.119*(C1+C2+C3)**.9
      C7= LP*(2.+.2168*CP+.377*CP**.7756)
      C9=51.*CP**.60979
      IF(CP.LE.25.) C9=87.*CP**.4476
      IF(CP.LE. 7.) C9=122.*CP**.25874
      IF(CP.LE.1.5) C9=132.*CP**.08392
      C9=C9*BLS1/3.76
      C10= BLS2/104.4*7.3*UF*CP
      XP=.58*CP**1.006
      C11= FR*445.3*XP**.986
      C12=64.*XP**.5476
      IF(XP.LE.40.) C12=70.*XP**.5
      IF(XP.LE.6.) C12=78.*XP**.44898
      IF(XP.LE.1.5) C12=80.*XP**.3571
      C12=C12*BLS1/3.76
      C13=EC*1.46*CP
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      C14=0.0
      CBR1=(1.-50./TDSI)/(900./CAI-1.)
      CBR2=(TDSI-50.)/(80000.-TDSI)
      IF(CBR1.LE.CBR2) BPR=CBR2
      IF(CBR1.GT.CBR2) BPR=CBR1
      CB=BPR*CP
      CW=CP*(2.-BPR)
      IF(ICTC.GE.1) CW=CP*(3.2-BPR)
      TDSP=50.
      GO TO 1000
C
C        ***  COST ANALYSIS FOR VC-VTE-MSF SYSTEMS  ***
C
    3 C1=ENRI/952.*2.71*CP**.7451
      IF(CP.LE.2.) C1=ENRI/952.*2.88*CP**.64706
      IF(CP.LE.1.) C1=ENRI/952.*2.88*CP**.51961
      C2=0.0
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C2)*IR/24.*(17.368+1.263*CP)
      C6=.119*(C1+C2+C3)**.9
      C7=LP*.8+LP*.2133*CP
      C9=46.*CP**.83077
      IF(CP.LE.30.) C9= 80.*CP**.6713
      IF(CP.LE. 6.) C9=111.*CP**.4797
      IF(CP.LE. 2.) C9=122.*CP**.36364
      IF(CP.LE. 1.) C9=122.*CP**.2797
      C9=C9*BLS1/3.76
      C10= BLS2/104.4*7.3*UF*CP
      C11=FR*151.475*CP
      C12=C13=0.
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      C14=0.
      CBR1=(1.-50./TDSI)/(900./CAI-1.)
      CBR2=(TDSI-50.)/(80000.-TDSI)
      IF(CBR1.LE.CBR2) BPR=CBR2
      IF(CBR1.GT.CBR2) BPR=CBR1
      CB=BPR*CP
      CW=CP*(1.5-BPR)
      IF(ICTC.GE.1) CW=CP*(2.2-BPR)
      TDSP=50.
      GO TO 1000
C
C        ***  COST ANALYSIS FOR ED SYSTEMS  ***
C
    4 HF=.575*((NAI+KI+CLI)/TDSI)+.014375*(TEMP-40.)
      FSR=.53/HF**.5418
      N=(ALOG10(TDSP)-ALOG10(TDSI))/(ALOG10(FSR))+1
      N1=CP/.252+1
      AS=N*N1
      TDSP=TDSI*FSR**N
      C1=ENRI/952.*.05194*AS**.84962
      IF(AS.LE.80.) C1=ENRI/952.*.08226*AS**.75188
      IF(AS.LE.25.) C1=ENRI/952.*.1503*AS**.56391
      IF(AS.LE.4.) C1=ENRI/952.*.203*AS**.37594
      C14=1.6706*AS**.9766
      C2=0.0
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C3)*IR/24.*8.*CP**.3137
      C6=.119*(C1+C2+C3)**.9
      C7=LP*(.0246*AS)
      C9=25.5*CP**.4726
      IF(CP.LE.1.5) C9=26.5*CP**.4144
      C9=HLS1/3.76* C9+10.*C1
      C10=HLS2/104.4*18.25*UF*CP
      C11=C12=0.
      C13=EC*(20.47/TEMP**.403*(TDSI/1000.)**1.0342)*CP*.365
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      BPR=(1.-TDSP/TDSI)/(900./CAI-1.)
      IF(BPR.LT.0.15) WRITE(6,103)
  103 FORMAT(1H ,48HED REMOVAL EFFICIENCY IS TOO HIGH-STOP ANALYSIS     )
      CB=BPR*CP
      CW=0.
      GO TO 1000
C
C        ***  COST ANALYSIS FOR RO SYSTEMS  ***
C
    5 XT=CP*(100.+(77.-TEMP)*1.7)/100.
      C1=ENRI/952.*.473*XT**.91
      IF(CP.LE.8.) C1=ENRI/952.*.522*XT**.85774
      IF(CP.LE.4.) C1=ENRI/952.*.575*XT**.79184
      C2=0.0
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C3)*IR/24.*8.*CP**.3137
      C6=.119*(C1+C2+C3)**.9
      C7=LP*(.5+.298*CP)
      C9=25.5*CP**.4726
      IF(CP.LE.1.5) C9=26.5*CP**.4144
      C9=HLS1/3.76* C9+10.*C1
      C10=HLS2/104.4*18.25*UF*CP
      C11=C12=0.0
      C13=EC*3.65*CP
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      C14=36.5*UF*CP
      BPR=(1.-TDSP/TDSI)/(900./CAI-1.)
      IF(BPR.LT.0.11) WRITE(6,111)
  111 FORMAT(1H ,48HRO REMOVAL EFFICIENCY IS TOO HIGH-STOP ANALYSIS    )
      CW=0.
      CB=BPR*CP
      GO TO 1000
C
C        ***  COST ANALYSIS FOR VC-VF SYSTEMS  **
C
    6 C1=ENRI/952.*1.77*CP**.8520
      C2=0.
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C3)*IR/24.*(17.368+1.263*CP)
      C6=.119*(C1+C2+C3)**.9
      C7=LP*(.8+0.32*CP)
      C9=132.*CP**.64624
      IF(CP.LE.7.) C9=132.*CP**.5986
      C9=C9*BLSI/3.76
      C10=C11=C12=0.0
      C13=EC*(10.+.2375*TEMP+.0084375*(TDSI/10.**3)**2)*CP*.365
      IF(CP.LT.0.5) C13=C13*.83/CP**.257
      C14=0.
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      BPR=(TDSI-TDSP)/(60000.-TDSI)
      CB=BPR*CP
      CW=0.
      GO TO 1000
C
C        ***  COST ANALYSIS FOR IX SYSTEMS  ***
C
    7 YP=.82*HCO3I+1.41*CLI+.81*NO3I+1.04*SO4I
      BAR=.82*HCO3I/YP
      C1=ENRI/952.*(.8572-.4595*BAR)*((TDSI-TDSP)*CP/10.**3)**.6979
      C2=0.
      C3=ENRI/952.*.1*CP**.6535
      C4=(C1+C3)*IR/24.*8.*CP**.3137
      C6=.119*(C1+C2+C3)**.9
      C7=LP*(.8+.32*CP)
      C9=25.5*CP**.4726
      IF(CP.LE.1.5) C9=26.5*CP**.4144
      C9=BLSI/3.76* C9+10.*C1
      C10=3.650*(.023-.01696*BAR)*(TDSI-TDSP)*CP
      C11=C12=0.0
      C13=EC*(1.0514*CP**.8533)*.365
      C14=30.*C1
      C5=0.08333*(C9+C10+C11+C12+C13)/1000.
      C8=2.*C5
      BPR=(TDSI-TDSP)/1000.
      CB=BPR*CP
      CW=0.
 1000 CONTINUE
      CT1=C9+C10+C11+C12+C13
      CT2=(C1+C2+C3+C4+C5+C6)*FCR*1000.0
      CT3=(C7+C8)*IR*1000.0
      CT4=CT2+CT3
      CT5=CT1+CT4+C14
      COW=CT5/CP/365.
      CBO=((CP+CB)*TDSI-CP*TDSP)/CB
      CUS=CT5*1000./CB/1.5234/CBO
      TONS=CB*CBO*1.5234
   11 CONTINUE
      RETURN
      END
```

FEEDWATER AND BRINE DISPOSAL COSTS

```
      SUBROUTINE ADJUST(ICTC,ENR1,CW,IFWC,CP,DEPTHF,PHI,CPI,DISTF,PPI,E,EI,IBC,D
     1EI,IBC,DEPTHB,LP,EC,BLS1,C1A,C2A,C3A,C4A,C5A,C6A,C7A,C8A,C9A,C10A,
     2C11A,C12A,C13A,C14A,DISTB,CB,I)
      REAL LP,I
C
C        COOLING TOWER CAPITAL COSTS
C
      C1CT=0.
      IF(ICTC.GE.1) C1CT=ENR1/952.*(.019*CW**.854)
C
C        FEEDWATER WELL CAPITAL COSTS INCLUDING SURFACE FACILITIES
C
      C1FW=0.
       ANFW=CP/5.+1.
      DEPTHF=DEPTHF/1000.
      IF(IFWC.EQ.2) GO TO 1
      C1FW=ANFW*ENR1/952.*((.3+.6875*DEPTHF)*.041*(CP/ANFW)**.4225)
     1+PHI/1.41*((1.6+.48*CP)+(1.4+1.316*CP))/1000.*ANFW
C
C        FEEDWATER PIPELINE CAPITAL COSTS
C
    1 CPCW=CP+CW
      IF(ICTC.EQ.2) CPCW=CP
      C1PL=CPI/1.17*3.4*CPCW**.7164*DISTF/10.**6
      IF(CPCW.LE.7.) C1PL=CPI/1.17*5.0*CPCW**.5442*DISTF/10.**6
      IF(CPCW.LE.2.) C1PL=CPI/1.17*5.8*CPCW**.3503*DISTF/10.**6
      IF(CPCW.LE..6) C1PL=CPI/1.17*5.5*CPCW**.2007*DISTF/10.**6
C
C        PIPELINE PUMPING PLANT CAPITAL COSTS
C
      H=DISTF/5280.*147./CPCW**.773
      IF(CPCW.LE.15.) H=DISTF/5280.*71./CPCW**.515
      IF(CPCW.LE. 4.) H=DISTF/5280.*58./CPCW**.339
      HK=.07163*H**.6548
      IF(H.LE.150.) HK=.1978*H**.4667
      IF(H.LE. 80.) HK= .6637*H**.178
      C1PPP=PPI/1.26*HK*5.8*CPCW**.9703/1000.
C
C        EVAPORATION POND CAPITAL COSTS
C
      EPA=1.12*CB/E
      C1EP=0.
      IF(IBC.EQ.2) C1EP=EI/1.27*6.333*EPA
C
C        BRINE INJECTION WELL AND SURFACE FACILITY CAPITAL COSTS
C
      DEPTHB=DEPTHB/1000.
      C1BW=0.
      IF(IBC.EQ.2) GO TO 10
      AK=.32+.3692*DEPTHB
      IF(DEPTHB.LE.6.) AK=.6+.3167*DEPTHB
      IF(DEPTHB.LE.2.) AK=.8+.2367*DEPTHB
      C1BW=ENR1/952.*AK*.153*CB**.9867
      IF(CB.LE.2.5) C1BW=ENR1/952.*AK*.192*CB**.748
      IF(CB.LE. .8) C1BW=ENR1/952.*AK*.188*CB**.4258
      IF(CB.LE. .3) C1BW=ENR1/952.*AK*.128*CB**.1011
      C1BM=     CPI/1.17*3.4*CB**.7164*DISTB/10.**6
      IF(CB.LE.7.) C1BM=     CPI/1.17*5.0*CB**.5442*DISTB/10.**6
      IF(CB.LE.2.) C1BM=     CPI/1.17*5.8*CB**.3503*DISTB/10.**6
      IF(CB.LE..6) C1BM=     CPI/1.1*5.5*CB**.2007*DISTB/10.**6
      C1BW=C1BW+C1BM
      C1BW1=.29*CB*PMI/1.41
      IF(CB.LE.5.) C1BW1=(.2+.2525*CB)*PMI/1.41
      IF(CB.LE.2.) C1BW1=(.3+.23*CB  )*PMI/1.41
      C1BW=C1BW+C1BW1
   10 CONTINUE
      C1A=C1CT+C1FW+C1PL+C1PPP+C1FP+C1BW
C
C        MISCELLANEOUS CAPITAL COSTS
C
      C2A=C3A=C5A=C10A=C11A=C12A=C14A=C8A=0.
      C4A=C1A*1/2.
      C6A=.119*C1A**.9
      A11=A12=A13=A14=A15=A16=0.
      IF(IFWC.NE.2) A11=ANFW/2.
      IF(ICTC.EQ.1) A12=1.+.0417*CW
      A13=DISTF*(1.5+.00625*CPCW)/5280.
      A14=CB/2.
      A15=EPA*1000.
      A16=DISTB*(1.5+.00625*CB)/5280.
      C7A=(A11+A12+A13+A14+A15+A16)*LP
C
C        COOLING TOWER O AND M
C
      OMCT=C1CT*10.
C
C        PIPELINE PUMPING PLANT O AND M
C
      OMPLPP=BLS1/3.76*1.68*CPCW**.7546
      IF(CPCW.LE.1.5) OMPLPP=BLS1/3.76*1.76*CPCW**.6557
      IF(CPCW.LE..25) OMPLPP=BLS1/3.76*1.44*CPCW**.5255
C
C        FEEDWATER WELL O AND M
C
      OMFW=0.
      IF(IFWC.EQ.2) GO TO 2
      OMFW=BLS1/3.76*21.*CP**.869
      IF(CP.LE.1.) OMFW=BLS1/3.76*21.*CP**.6818
      IF(CP.LE..25) OMFW=BLS1/3.76*16.*CP**.4982
C
C        INJECTION WELL O AND M
C
    2 OMBW=0.
      IF(IBC.EQ.2) GO TO 3
      OMBW=BLS1/3.76*44.*CB**.8728
      IF(CB.LE.2.) OMBW=BLS1/3.76*50.*CB**.7644
      IF(CB.LE..5) OMBW=BLS1/3.76*44.*CB**.4479
C
C        EVAPORATION POND O AND M
C
    3 OMEP=.0075*C1EP
      C9A=OMCT+OMPLPP+OMFW+OMBW+OMEP
C
C        MISCELLANEOUS O AND M
C
      Y=147./CPCW**.773
      IF(CPCW.LE.15.) Y=71./CPCW**.515
      IF(CPCW.LE. 4.) Y=58./CPCW**.339

      HEADF=(DISTF*Y/5280.+200.)*CPCW*1000.
      Y=147./CB**.773
      IF(CB.LE.15.) Y=71./CB**.515
      IF(CB.LE. 4.) Y=58./CB**.339
      HEADB=(DISTB*Y/5280.+200.)*CB*1000.
      C13A=.004*(HEADF+HEADB)*EC*365./10.**6
      C8A=(C9A+C13A)/6000.
      DEPTHF=DEPTHF*1000.
      DEPTHB=DEPTHB*1000.
      RETURN
      END
```

LISTING OF OUTPUT SUBROUTINE

```
      SUBROUTINE OUTPUT1(A1,A2,A3,44,A5,A6,A7,A8,A9,A10,A11,A12, A13,A14
     1,A15,A16,A17,A18,NAME,K)
      DIMENSION A1(8),A2(8),A3(8),A4(8),A5(8),A6(8),A7(8),A8(8),A9(8),
     1A10(8),A11(8),A12(8),A13(8),A14(8),A15(8),A16(8),A17(8),A18(8),
     2NAME(7,10)
      DIMENSION R(8),R1(8),R2(8)
      WRITE(6,100) (NAME(K,J),J=1,10)
  100 FORMAT(1H1,10A8//)
      WRITE(6,101)
  101 FORMAT(1H ,24X,1H*,25X,26HDESALTING PLANT CAPACITIES,20X,1H*)
      WRITE(6,102)
  102 FORMAT(1H ,24X,1H*,75H-----------------------------------------------
     1---------------------------*               )
      WRITE(6,103)
 103  FORMAT(1H ,24X,1H*,3X,3H.25,6X,3H.50,6X,3H1.0,6X,3H2.0,6X,3H4.0,
     16X,3H8.0,5X,4H16.0,5X,7H32.0   *)
      WRITE(6,104)
  104 FORMAT(1H ,4X,16HCOST DESCRIPTION,4X,1H*,3X,3HMGD,6X,3HMGD,6X,
     13HMGD,6X,3HMGD,6X,3HMGD,6X,3HMGD,6X,3HMGD,6X,6HMGD   *)
      WRITE(6,105)
  105 FORMAT(1H ,100H--------------------------------------------------
     1----------------------------------------------*         )
      WRITE(6,106)
  106 FORMAT(1H ,25HCAPITAL COSTS/10**6      *,71X,1H*)
      WRITE(6,107)
  107 FORMAT(1H ,25H A. CONSTRUCTION         *)
      WRITE(6,108) (A1(I),I=1,8)
  108 FORMAT(1H+,26X,F6.2,3X,F6.2,3X,F6.2,3X,F6.2,3X,F6.2,3X,F6.2,3X,F6.
     12,3X,F6.2,2H *)
      WRITE(6,109)
  109 FORMAT(1H ,25H B. STEAM FACILITIES     *)
      WRITE(6,108) (A2(I),I=1,8)
      WRITE(6,110)
  110 FORMAT(1H ,25H C. SITE DEVELOPMENT     *)
      WRITE(6,108) (A3(I),I=1,8)
      WRITE(6,111)
  111 FORMAT(1H ,25H D. INTEREST-(CONSTR)    *)
      WRITE(6,108) (A4(I),I=1,8)
      WRITE(6,112)
  112 FORMAT(1H ,25H E. START-UP             *)
      WRITE(6,108) (A5(I),I=1,8)
      WRITE(6,113)
  113 FORMAT(1H ,25H F. GENERAL EXPENSE      *)
      WRITE(6,108) (A6(I),I=1,8)
      WRITE(6,114)
  114 FORMAT(1H ,25H G. LAND                 *)
      WRITE(6,108) (A7(I),I=1,8)
      WRITE(6,115)
  115 FORMAT(1H ,25H H. WORKING CAPITAL      *)
      WRITE(6,108) (A8(I),I=1,8)
      WRITE(6,116)
  116 FORMAT(1H ,24X,1H*,75H-----------------------------------------------
     1---------------------------*      )
      X1=A1(1)+A2(1)+A3(1)+A4(1)+A5(1)+A6(1)+A7(1)+A8(1)
      X2=A1(2)+A2(2)+A3(2)+A4(2)+A5(2)+A6(2)+A7(2)+A8(2)
      X3=A1(3)+A2(3)+A3(3)+A4(3)+A5(3)+A6(3)+A7(3)+A8(3)
      X4=A1(4)+A2(4)+A3(4)+A4(4)+A5(4)+A6(4)+A7(4)+A8(4)
      X5=A1(5)+A2(5)+A3(5)+A4(5)+A5(5)+A6(5)+A7(5)+A8(5)
      X6=A1(6)+A2(6)+A3(6)+A4(6)+A5(6)+A6(6)+A7(6)+A8(6)
      X7=A1(7)+A2(7)+A3(7)+A4(7)+A5(7)+A6(7)+A7(7)+A8(7)
      X8=A1(8)+A2(8)+A3(8)+A4(8)+A5(8)+A6(8)+A7(8)+A8(8)
      WRITE(6,117)
  117 FORMAT(1H ,25H                  SUBTOTAL *)
      WRITE(6,108) X1,X2,X3,X4,X5,X6,X7,X8
      WRITE(6,118)
  118 FORMAT(1H ,24X,1H*,71X,1H*)
      WRITE(6,119)
  119 FORMAT(1H ,25HANNUAL COSTS/10**3      *,71X,1H*)
      WRITE(6,120)
  120 FORMAT(1H ,25H I. LABOR-MATERIALS     *)
      WRITE(6,132) (A9(I),I=1,8)
      WRITE(6,121)
  121 FORMAT(1H ,25H J. CHEMICALS           *)
      WRITE(6,132) ( A10(I),I=1,8)
      WRITE(6,122)
  122 FORMAT(1H ,25H K. FUEL                *)
      WRITE(6,132) ( A11(I),I=1,8)
      WRITE(6,123)
  123 FORMAT(1H ,25H L. STEAM               *)
      WRITE(6,132) ( A12(I),I=1,8)
      WRITE(6,124)
  124 FORMAT(1H ,25H M. ELECTRICITY         *)
      WRITE(6,132) ( A13(I),I=1,8)
      WRITE(6,125)
  125 FORMAT(1H ,25H N. REPLACEMENT         *)
      WRITE(6,132) ( A14(I),I=1,8)
      WRITE(6,116)
      WRITE(6,117)
      Y1=A9(1)+A10(1)+A11(1)+A12(1)+A13(1)+A14(1)
      Y2=A9(2)+A10(2)+A11(2)+A12(2)+A13(2)+A14(2)
      Y3=A9(3)+A10(3)+A11(3)+A12(3)+A13(3)+A14(3)
      Y4=A9(4)+A10(4)+A11(4)+A12(4)+A13(4)+A14(4)
      Y5=A9(5)+A10(5)+A11(5)+A12(5)+A13(5)+A14(5)
      Y6=A9(6)+A10(6)+A11(6)+A12(6)+A13(6)+A14(6)
      Y7=A9(7)+A10(7)+A11(7)+A12(7)+A13(7)+A14(7)
      Y8=A9(8)+A10(8)+A11(8)+A12(8)+A13(8)+A14(8)
      WRITE(6,132) Y1,Y2,Y3,Y4,Y5,Y6,Y7,Y8
  132 FORMAT(1H+,26X,F6.1,3X,F6.1,3X,F6.1,3X,F6.1,3X,F6.1,3X,F6.1,3X,F6.
     11,3X,F6.1,2H *)
      WRITE(6,118)
      WRITE(6,118)
      WRITE(6,126)
  126 FORMAT(1H ,25HTOTAL ANNUAL COSTS      *,71X,1H*)
      WRITE(6,127)
  127 FORMAT(1H ,25H O. ANNUAL COSTS/10**3  *,71X,2H* )
      WRITE(6,131) ( A15(I),I=1,8)
      WRITE(6,128)
  128 FORMAT(1H ,25H P. WATER COST-$/1000 G *)
      WRITE(6,108) ( A16(I),I=1,8)
      WRITE(6,129)
  129 FORMAT(1H ,25H Q. SALT COSTS-$/TON    *)
      WRITE(6,108) ( A17(I),I=1,8)
      WRITE(6,118)
      WRITE(6,130)
  130 FORMAT(1H ,25HSALT REMOVED (TONS)     *)
      WRITE(6,131) (A18(I),I=1,8)
  131 FORMAT(1H+,25X,F7.0,2X,F7.0,2X,F7.0,2X,F7.0,2X,F7.0,2X,F7.0,2X,
     1F7.0,2X,F7.0,2H *)
```

```
      C=A17(8)
      B=1.
      DO 10 I=1,10
      A=(A17(4)-C)*2.**B
      B=ALOG10(A/(A17(1)-C))/(-0.6020599913)
   10 C=A17(8)-A/32.**B
      WRITE(6,133) A,B,C
  133 FORMAT(1H ,///,10X,21HSALT COSTS ($/TON) = ,F10.5,19H/(CAPACITY (M
     1GD))**,F10.7, 5H  +  ,F10.5   )
      DO 11 I=1,8
      R(I)=A16(I)/3.78541
      R1(I)=A17(I)/.9072
   11  R2(I)=A18(I)*.9072
      WRITE(6,134)
  134 FORMAT(1H0,////,25H SUMMARY IN METRIC UNITS   )
      WRITE(6,118)
      WRITE(6,135)
  135 FORMAT(1H ,24X,1H*,22X,40HDESALTING PLANT CAPACITIES
     1          ,9X,1H*)
      WRITE(6,102)
      WRITE(6,136)
  136 FORMAT(1H ,24X,1H*,3X,3H.95,5X,4H1.89,5X,4H3.79,5X,4H7.57,4X,5H15.
     114,4X,5H30.28,4X,5H60.57,3X,6H121.13,3H  *)
      WRITE(6,118)
      WRITE(6,199)
  199 FORMAT(1H ,44X,33HTHOUSANDS OF CUBIC METERS PER DAY  ,19X,1H*)
      WRITE(6,105)
      WRITE(6,126)
      WRITE(6,137)
  137 FORMAT(1H ,25H P. WATER COST-$/M**3   *)
      WRITE(6,108) (R(I),I=1,8)
      WRITE(6,138)
  138 FORMAT(1H ,25H Q. SALT COSTS-$/MTON   *)
      WRITE(6,108) (R1(I),I=1,8)
      WRITE(6,118)
      WRITE(6,139)
  139 FORMAT(1H ,25HSALT REMOVED (MTONS)    *)
      WRITE(6,131) (R2(I),I=1,8)
      RETURN
      END
```

LISTING INPUT DUMP SUBROUTINE

```
      SUBROUTINE WRITE(CP,UF,ENRI,BLS1,BLS2,FCR,IR
     1,TEMP,TDSI,NAI,KI,HCO3I,NO3I,SO4I,CLI,CAI,TDSO,EC,FR,LP,PMI,CPI,EI
     1,DEPTHF,DISTF,DEPTHB,DISTB,IFWC,IBC,E,PPI)
      REAL NAI,KI,NO3I,IR,LP
      WRITE(6,100)
  100 FORMAT(1H1,///,23HDESALTING COST ANALYSIS  )
      WRITE(6,101)
  101 FORMAT(1H0,6X,10HINPUT DATA//76H************************************
     1****************************************   /)
      WRITE(6,102)
  102 FORMAT(1H ,16HPLANT VARIABLES )
      WRITE(6,130)
  130 FORMAT(1H+,30X,29HFEEDWATER AND BRINE VARIABLES  )
      WRITE(6,103) CP
  103 FORMAT(1H ,1X,12H1. CAPACITY=,F5.1,4H MGD)
      WRITE(6,131) PMI
  131 FORMAT(1H+,31X,28H1. PRIME MOVER COST INDEX =   ,F6.2)
      WRITE(6,104) UF
  104 FORMAT(1H ,1X,14H2. USE FACTOR=,F5.2)
      WRITE(6,132) CPI
  132 FORMAT(1H+,31X,30H2. CONCRETE PIPE COST INDEX = ,F6.2)
      WRITE(6,105) ENRI
  105 FORMAT(1H ,1X,18H3. ENR BLDG INDEX=,F6.1)
      WRITE(6,133) PPI
  133 FORMAT(1H+,31X,40H3. PIPELINE PUMPING PLANT COST INDEX =  ,F6.2)
      WRITE(6,106) BLS1
  106 FORMAT(1H ,1X,19H4. BLS LABOR INDEX=,F5.2)
      WRITE(6,134) EI
  134 FORMAT(1H+,31X,26H4. EARTHWORK COST INDEX =  ,F6.2)
      WRITE(6,107) BLS2
  107 FORMAT(1H ,1X,18H5. BLS CHEM INDEX=,F6.1)
      IF(IFWC.EQ.2) WRITE(6,135)
  135 FORMAT(1H+,31X,41H5. SOURCE OF FEEDWATER- SURFACE DIVERSION   )
      IF(IFWC.EQ.1) WRITE(6,136)
  136 FORMAT(1H+,31X,29H5. SOURCE OF FEEDWATER- WELLS   )
      WRITE(6,108) FCR
  108 FORMAT(1H ,1X,16H6. FIX CHG RATE=,F6.5)
      IF(IFWC.EQ.1) WRITE(6,137) DEPTHF
  137 FORMAT(1H+,31X,24H6. AVERAGE WELL DEPTH =  ,F6.0, 3H FT)
      WRITE(6,109) IR
  109 FORMAT(1H ,1X,12H7. INT RATE=,F5.2)
      IF(IFWC.EQ.1) WRITE(6,138) DISTF
  138 FORMAT(1H+,31X,33H7. AVERAGE DISTANCE FROM PLANT = ,F6.0,3H FT)
      WRITE(6,110)
  110 FORMAT(1H ,34X)
      IF(IBC.NE.1) WRITE(6,140)
  140 FORMAT(1H+,31X,44H8. TYPE OF BRINE DISPOSAL- EVAPORATION PONDS   )
   10 IF(IBC.EQ.1) WRITE(6,139)
  139 FORMAT(1H+,31X,42H8. TYPE OF BRINE DISPOSAL- INJECTION WELLS  )
      WRITE(6,111)
  111 FORMAT(1H ,21HWATER CHARACTERISTICS)
      IF(IBC.NE.1) WRITE(6,141) E
  141 FORMAT(1H+,31X,29H9. ANNUAL EVAPORATION RATE = ,F6.0,3H FT)
      IF(IBC.EQ.1) WRITE(6,142) DEPTHB
  142 FORMAT(1H+,31X,24H9. AVERAGE WELL DEPTH =  ,F6.0, 3H FT)
      WRITE(6,112) TEMP
  112 FORMAT(1H ,1X,21H1. FEED TEMP (DEG F)=,F5.2)
      IF(IBC.EQ.1) WRITE(6,143) DISTB
  143 FORMAT(1H+,31X,33H10. AVERAGE DISTANCE FROM PLANT = ,F6.0,3H FT)
```

```
      WRITE(6,113) TDSI
  113 FORMAT(1H ,1X,18H2. FEED TDS (PPM)=,F5.0)
      WRITE(6,114) NAI
  114 FORMAT(1H ,1X,17H3. FEED NA (PPM)=,F5.0)
      WRITE(6,115) KI
  115 FORMAT(1H ,1X,16H4. FEED K (PPM)=,F5.0)
      WRITE(6,116) HCO3I
  116 FORMAT(1H ,1X,19H5. FEED HCO3 (PPM)=,F5.0)
      WRITE(6,117) NO3I
  117 FORMAT(1H ,1X,18H6. FEED NO3 (PPM)=,F5.0,10X)
      WRITE(6,118) SO4I
  118 FORMAT(1H ,1X,18H7. FEED SO4 (PPM)=,F5.0)
      WRITE(6,119) CLI
  119 FORMAT(1H ,1X,17H8. FEED CL (PPM)=,F5.0)
      WRITE(6,120) CAI
  120 FORMAT(1H ,1X,17H9. FEED CA (PPM)=,F5.0)
      WRITE(6,121) TDSO
  121 FORMAT(1H ,   22H10. PRODUCT TDS (PPM)=,F5.0)
      WRITE(6,122)
  122 FORMAT(1H )
      WRITE(6,123)
  123 FORMAT(1H ,13HMISCELLANEOUS)
      WRITE(6,124) EC
  124 FORMAT(1H ,1X,13H1. ELEC RATE=,F9.5)
      WRITE(6,125)
  125 FORMAT(1H ,4X,16H    ($/1000 KWH))
      WRITE(6,126) FR
  126 FORMAT(1H ,1X,13H2. FUEL RATE=,F9.5)
      WRITE(6,127)
  127 FORMAT(1H ,4X,17H    ($/MBTU)     )
      WRITE(6,128) LP
  128 FORMAT(1H ,1X,14H3. LAND PRICE=, F9.6)
      WRITE(6,129)
  129 FORMAT(1H ,4X,15H($MILLION/ACRE))
      RETURN
      END
```

APPENDIX B

OPTIMIZATIONAL ANALYSIS COMPUTER CODE

DESCRIPTION OF CODE

Although the theory encompassing this optimization technique is a very powerful one, the computer code of the method has certain inherent limitations. This is not a fault of the particular program, but rather a characteristic of nearly all programs with any degree of sophistication. The utility of any optimum seeking procedure in engineering applications is largely dependent on the economy of use and its generality. It is primarily the latter aspect that limits the subsequent use by an individual unfamiliar with the mechanics of the programs' operation. Very few large computer programs are general enough to be used with little or no knowledge of their structure and weak points. The computer code developed in this section is not among these very few, but a great deal of time and effort has been spent in maximizing the generality of the program.

The Jacobian Differential Algorithm consists of a main program and routines using common and call data transfers. A summary of the role of each subroutine is given in Table B-1. The entire system can be subdivided into seven groups according to their role in the optimizing technique:

1. Problem definition is accomplished in subroutine CONTROL;
2. Input-Output is provided by the subroutines DATAOUT and ANSOUT;
3. The coordination of the entire program procedure is handled in subroutine DIFALGO;
4. Organization functions in the program are completed in REORGA and ARRAY;
5. Special computational subroutines include JORK, JACOB1, ENDCHEK, CONDER, KUNTUK, NEWTSIM, and GAUSS;
6. The principal parts of the program are encompassed in subroutine DECDJ, INCDJ, and INCFT which accomplish the step-by-step movement toward the optimum; and
7. The calculation of the constrained derivatives is done in the subroutine, KODRIV.

Although each of these subroutines have certain independent functions, it is probably only worthwhile to describe a select few so the reader can observe the basic operation of the program. A flow chart of the basic organization structure encompassed in DIFALGO is shown Figure B-1. Similar flow charts for selecting state and decision variables (REORGA), solution of system of non-linear equations by a Newton-Raphson (NEWTSIM), and the numerical change in the decision variables (DECDJ) are given in Figures B-2 through B-4, respectively.

The main program contains all input data requirements and serves only as an input-output system. The user supplies a problem title and subtitle, name, and a series of control variables:

(1) number of original variables, free variables, equality constraints, inequality constraints, type of objective function (linear or non-linear), and type of constraints (linear or non-linear); and
(2) maximum number of iterations toward the minimum, frequency of output of intermediate calculations (debugging output), and input dump controls.

Next to be read in are convergence tolerances, the characteristics of each variable (free or non-free variables), and the array of values associated with the x-variables in the objective function representing a feasible solution. The program returns a single value for the optimum which the user may output as desired. Usually, however, the main program should print the values of the independent variables and the resulting objective function. A formal output of the final solution is provided by subroutine ANSOUT. A summary of the input variables required and the definitions required to set up a problem is given in Table B-2. To address this subroutine, any even numbered integer can be listed in the argument of the CALL DIFALGO ( ) statement.

The program requires about 56k of core storage and will solve most problems within 10-20 central processor seconds. The existing common statement structure will handle a problem up to 30 variables and 30 constraints. A complete listing of the main program and subroutines is given in the following pages. Code language is Fortran IV and uses no tape or disk systems.

TABLE B-1. DEFINITION OF SUBROUTINE FUNCTIONS

Subroutine	Function
ANSOUT	Output of the optimal solution
ARRAY	Determination of initial variable partition
DATAOUT	Output of input data and control variables
DECDJ	Decreases the value of a decision variable
DIFALGO	Coordination of the complete algorithm
ENDCHEK	Checks problem to insure the search remains in a feasible region
GAUSS	Gaussian elimination procedure for solving system of linear equations
INCDJ	Increases the value of a decision variable
INCFT	Loosens a previouly active constraint
JACOB1	Computation of the determinant of the Jacobian matrix
JORK	Selection of the decision or slack variable resulting in the most decrease in the value of the objective function
KODRIV	Constrained derivatives, $\delta\phi_{\ell}^{+}/\delta d_p$, $\delta\phi_{\ell}^{+}/\delta\phi_p$, $\delta s_i/d_p$, $\delta s_i/\delta\phi_p$, $\delta y/\delta d_j$, and $\delta y/\delta\phi_j$
KUNTUK	Checks Kuhn-Tucker conditions for a minimum
NEWTSIM	Newton-Raphson method for solving systems of non-linear equations
CONTROL	Computes the value of the objective function, constraints, objective function derivatives, and constraint derivatives.

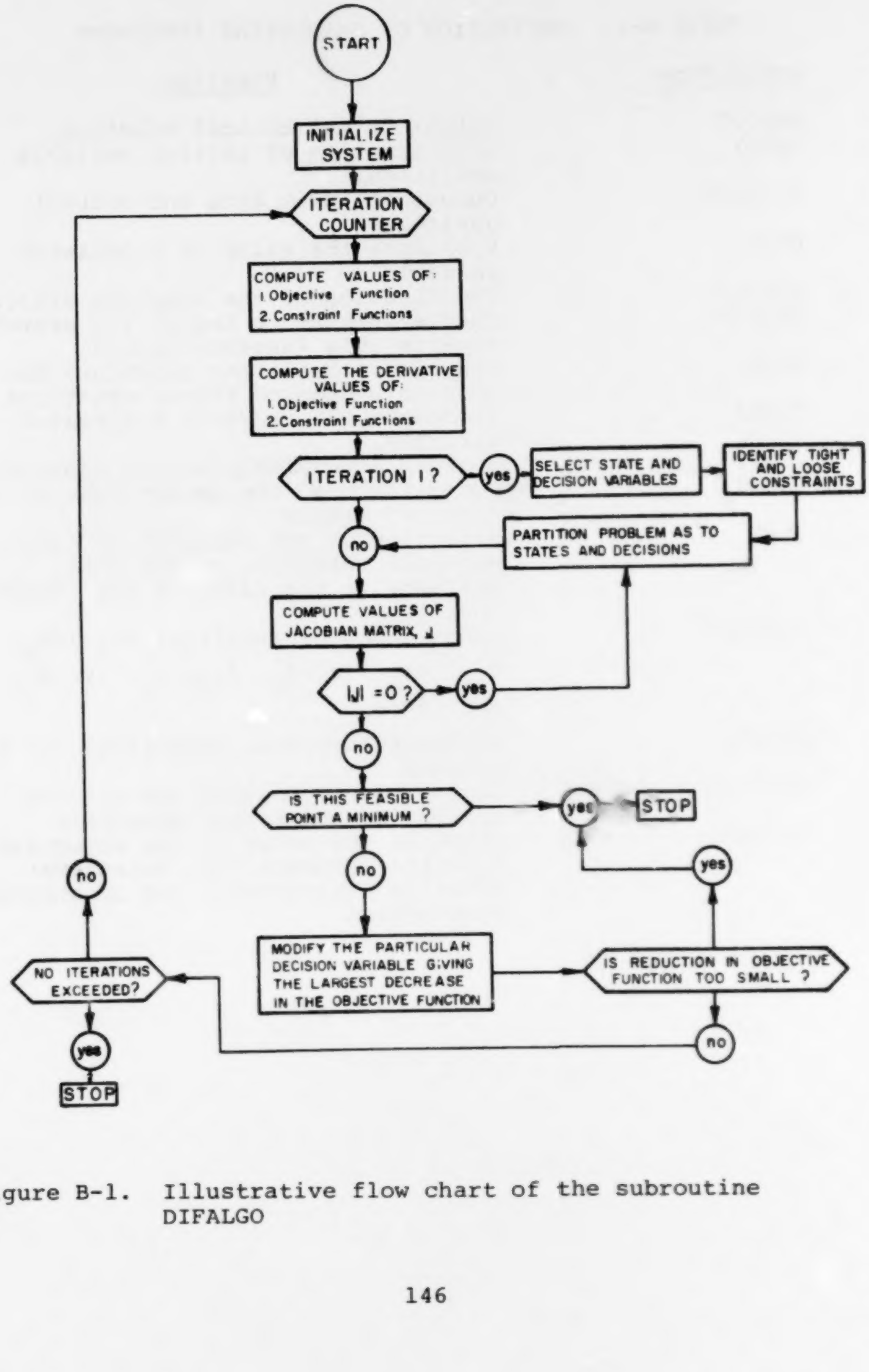
START
INITIALIZE SYSTEM
ITERATION COUNTER
COMPUTE VALUES OF: 1. Objective Function 2. Constraint Functions
COMPUTE THE DERIVATIVE VALUES OF: 1. Objective Function 2. Constraint Functions
ITERATION 1 ?
yes
SELECT STATE AND DECISION VARIABLES
IDENTIFY TIGHT AND LOOSE CONSTRAINTS
PARTITION PROBLEM AS TO STATES AND DECISIONS
no
COMPUTE VALUES OF JACOBIAN MATRIX, J
|J| = 0 ?
yes
no
IS THIS FEASIBLE POINT A MINIMUM ?
yes
STOP
no
yes
MODIFY THE PARTICULAR DECISION VARIABLE GIVING THE LARGEST DECREASE IN THE OBJECTIVE FUNCTION
IS REDUCTION IN OBJECTIVE FUNCTION TOO SMALL ?
no
NO ITERATIONS EXCEEDED?
no
yes
STOP

Figure B-1. Illustrative flow chart of the subroutine DIFALGO

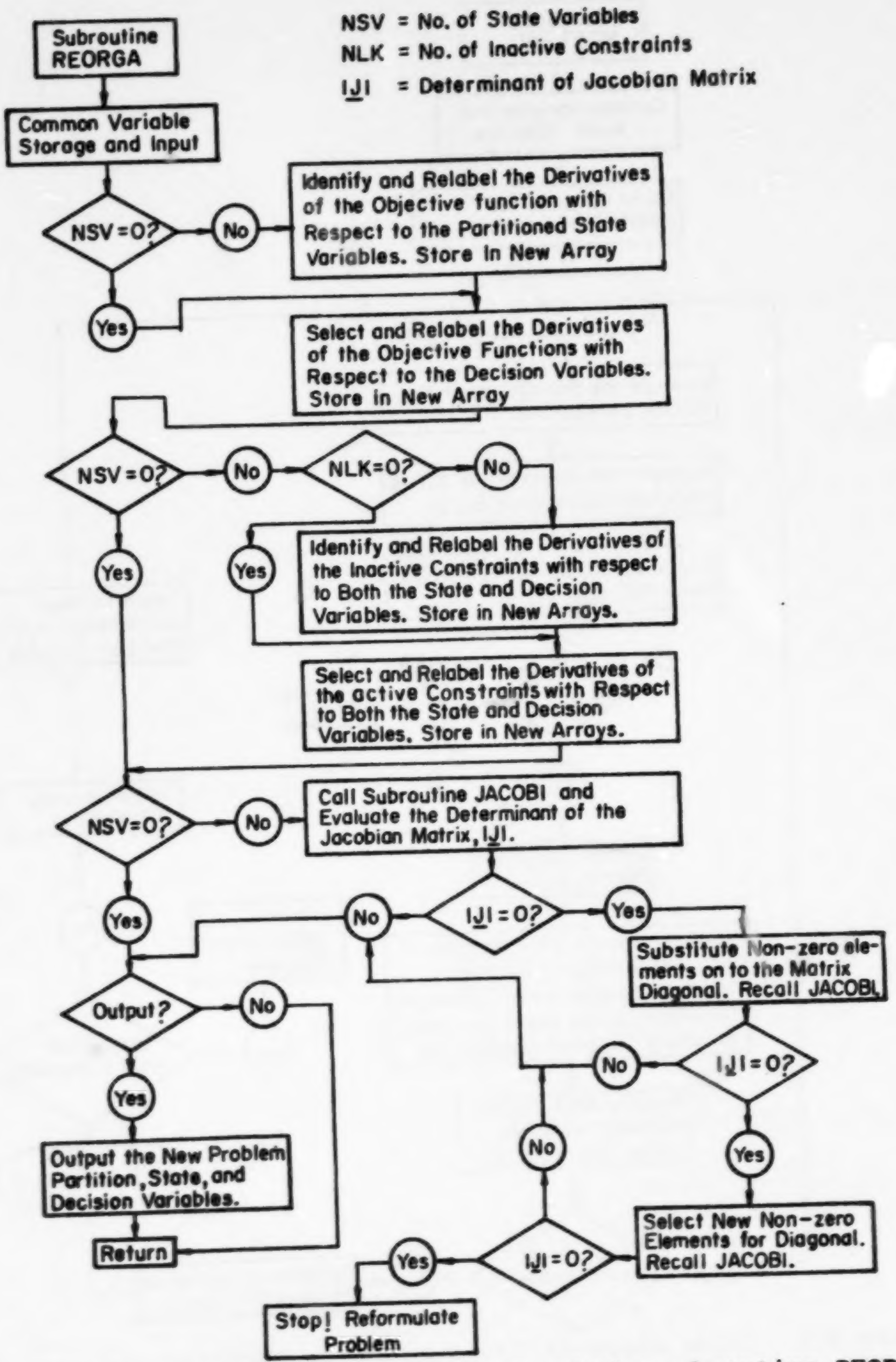
NSV = No. of State Variables
NLK = No. of Inactive Constraints
|J| = Determinant of Jacobian Matrix
Subroutine REORGA
Common Variable Storage and Input
NSV=0?
No
Yes
Identify and Relabel the Derivatives of the Objective function with Respect to the Partitioned State Variables. Store In New Array
Select and Relabel the Derivatives of the Objective Functions with Respect to the Decision Variables. Store in New Array
NSV=0?
No
NLK=0?
No
Yes
Yes
Identify and Relabel the Derivatives of the Inactive Constraints with respect to Both the State and Decision Variables. Store in New Arrays.
Select and Relabel the Derivatives of the active Constraints with Respect to Both the State and Decision Variables. Store in New Arrays.
NSV=0?
No
Call Subroutine JACOBI and Evaluate the Determinant of the Jacobian Matrix, |J|.
Yes
No
|J|=0?
Yes
Substitute Non-zero elements on to the Matrix Diagonal. Recall JACOBI
Output?
No
Yes
No
|J|=0?
Output the New Problem Partition, State, and Decision Variables.
No
Yes
Return
Select New Non-zero Elements for Diagonal. Recall JACOBI.
Yes
|J|=0?
Stop! Reformulate Problem

Figure B-2. Illustrative flow chart of the subroutine REORGA

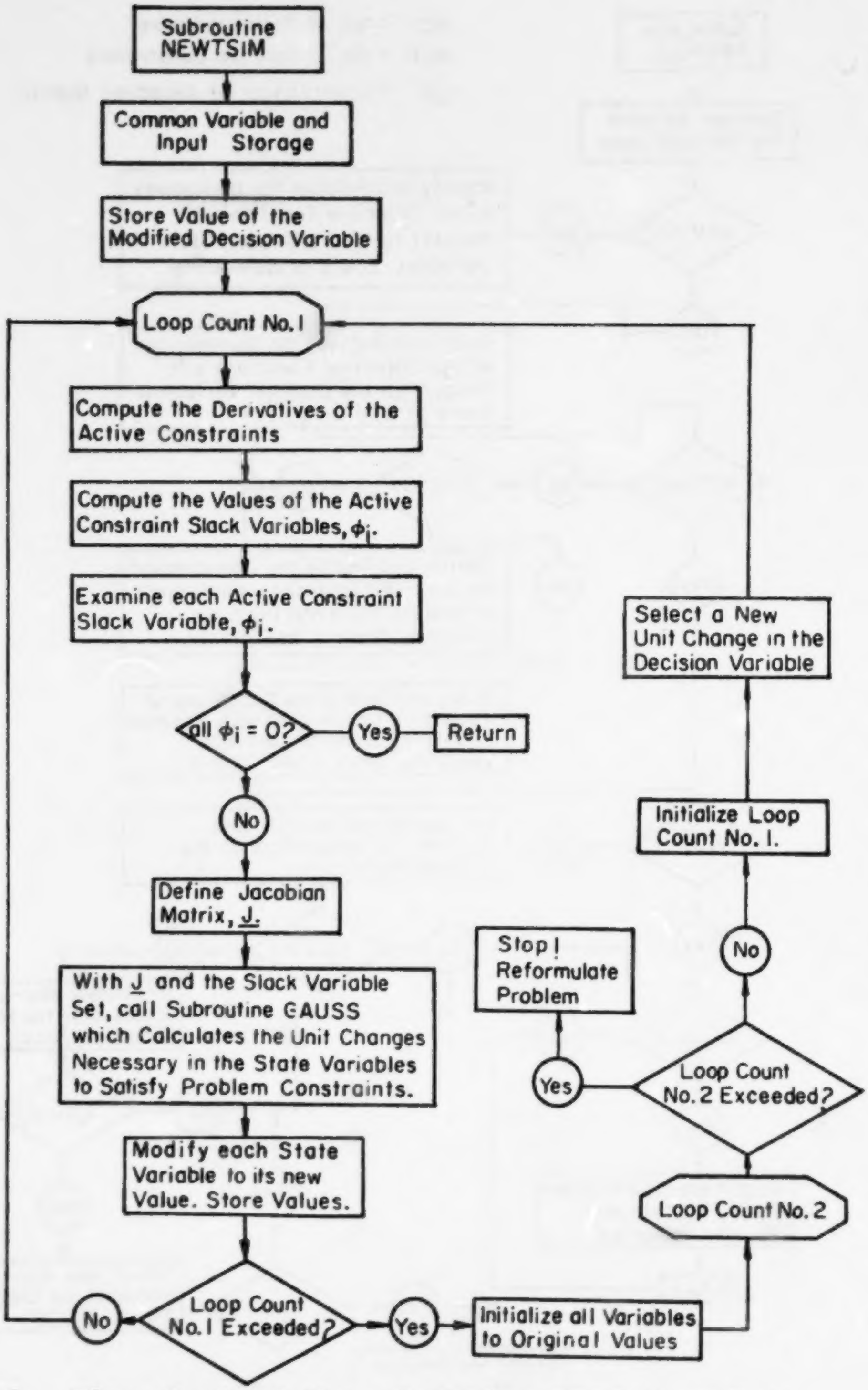
Subroutine NEWTSIM
Common Variable and Input Storage
Store Value of the Modified Decision Variable
Loop Count No. 1
Compute the Derivatives of the Active Constraints
Compute the Values of the Active Constraint Slack Variables, ϕ_i.
Examine each Active Constraint Slack Variable, ϕ_i.
all $\phi_i = 0$?
Yes
Return
No
Define Jacobian Matrix, $\underline{J}$.
With $\underline{J}$ and the Slack Variable Set, call Subroutine GAUSS which Calculates the Unit Changes Necessary in the State Variables to Satisfy Problem Constraints.
Modify each State Variable to its new Value. Store Values.
Loop Count No. 1 Exceeded?
No
Yes
Initialize all Variables to Original Values
Loop Count No. 2
Loop Count No. 2 Exceeded?
Yes
Stop! Reformulate Problem
No
Initialize Loop Count No. 1.
Select a New Unit Change in the Decision Variable

Figure B-3. Flow chart of the subroutine NEWTSIM used to solve systems of non-linear equations.

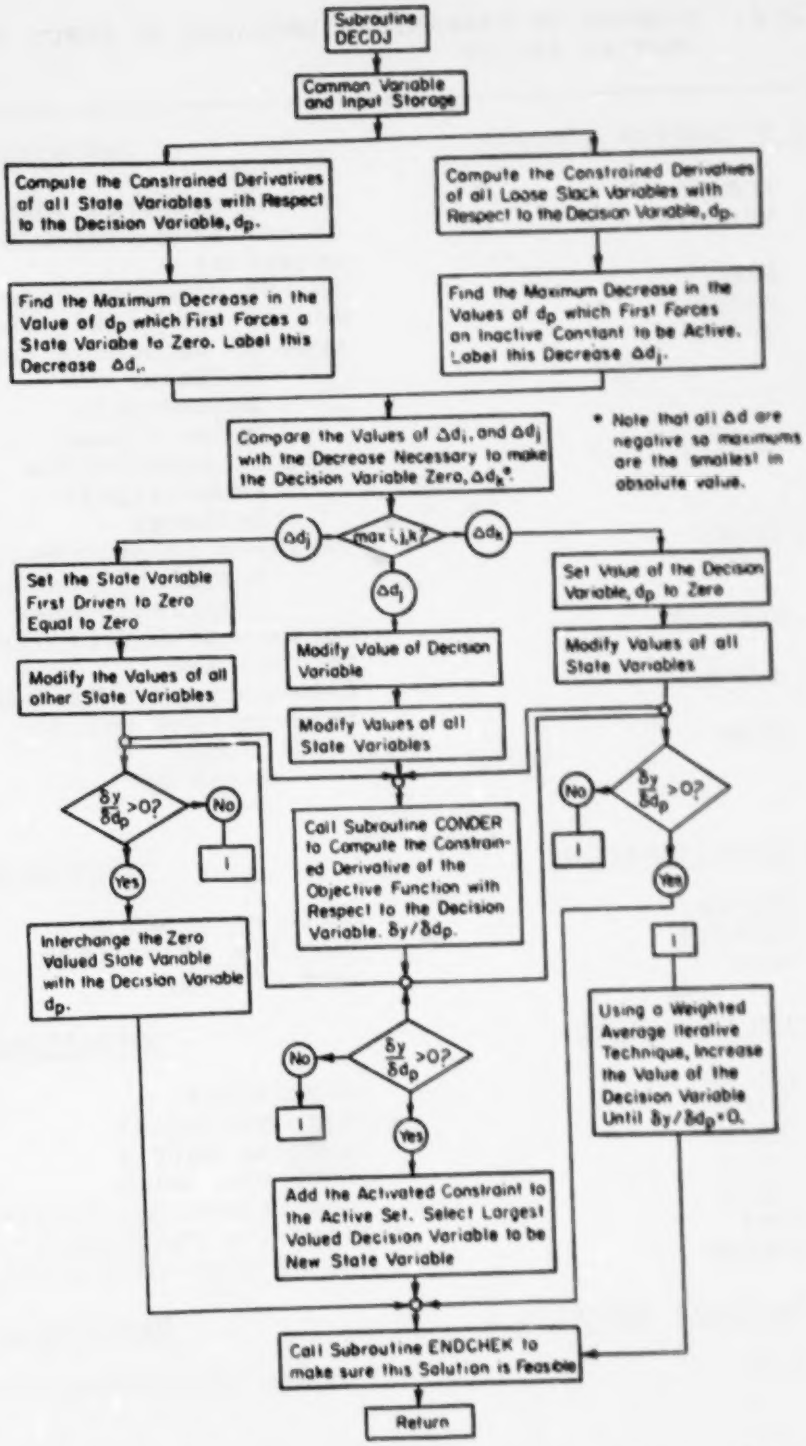
Subroutine DECDJ
Common Variable and Input Storage
Compute the Constrained Derivatives of all State Variables with Respect to the Decision Variable, d_p.
Compute the Constrained Derivatives of all Loose Slack Variables with Respect to the Decision Variable, d_p.
Find the Maximum Decrease in the Value of d_p which First Forces a State Variabe to Zero. Label this Decrease Δd_i.
Find the Maximum Decrease in the Values of d_p which First Forces an Inactive Constant to be Active. Label this Decrease Δd_j.
Compare the Values of Δd_i, and Δd_j with the Decrease Necessary to make the Decision Variable Zero, Δd_k*.
* Note that all Δd are negative so maximums are the smallest in absolute value.
max i, j, k?
Δd_j
Δd_i
Δd_k
Set the State Variable First Driven to Zero Equal to Zero
Modify the Values of all other State Variables
Modify Value of Decision Variable
Modify Values of all State Variables
Set Value of the Decision Variable, d_p to Zero
Modify Values of all State Variables
$\frac{\delta y}{\delta d_p} > 0?$
No
Yes
I
Call Subroutine CONDER to Compute the Constrained Derivative of the Objective Function with Respect to the Decision Variable. $\delta y / \delta d_p$.
Interchange the Zero Valued State Variable with the Decision Variable d_p.
Using a Weighted Average Iterative Technique, Increase the Value of the Decision Variable Until $\delta y / \delta d_p = 0$.
Add the Activated Constraint to the Active Set. Select Largest Valued Decision Variable to be New State Variable
Call Subroutine ENDCHEK to make sure this Solution is Feasible
Return

Figure B-4. Illustrative flow chart of the subroutine DECDJ

TABLE B-2. SUMMARY OF PARAMETERS REQUIRED AS INPUT AND PROBLEM SET-UP

Control Variables (Input)	Definition
NORIVA	no. of original variables
NFREEVA	no. of free variables (can be negative)
NKEQ	no. of equality constraints
NKINEQ	no. of inequality constraints
IYOFX	type of objective function 1=linear 2=quadratic 3=non-linear
ICTYPE	type of constraints 0=non-linear 1=linear
IFREE	variable identification 0=variable can be < 0 1=variable must be $\geq$ 0
MAXITER	maximum number of iterations per problem
MAXLEV	frequency of intermediate calculations output
ICON	input dump 1=yes 0=no

Problem Identification	Definition
ITITLE	title of problem
ISUBTIT	subtitle of problem
NAME	name of user

Convergence Tolerance	Definition
TOLCON	Constraints
TOLONS	State variables
TOLJAC	Jacobian Matrix
TOLVJ	Slack variables
TOLDJ	Constrained Derivatives
TOLY	Objective Function
TOLKUN	Kuhn-Tucker Conditions

Initial Feasible Solution	Definition
X(I)	Problem independent variable

(continued)

TABLE B-2. (continued)

Set-up Parameters (in subroutine control)	Definitions
Y	Value of objection function
ADYDX(I)	dy/dx
ADFDX(I,J)	dfi/dxj (f=constraints AFD(I)
AF(J)	Value of constraints

MAIN PROGRAM LISTING

```
       PROGRAM DASSL
      1(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
       COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
      1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
       COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
       COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
      1ND(30),NS(30),KT(30),KL(30),IFREE(30)
       COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
      10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
       COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
      1CODFLDD(30,30),CODFLDF(30,30)
       COMMON /MISC/ W(30),V(30),AMA(30,30)
       DIMENSION ITITLE(8),ISUBTIT(8),NAME(8)
       READ(5,10) (ITITLE(I),I=1,8)
    10 FORMAT(8A10)
       READ(5,10) (ISUBTIT(I),I=1,8)
       READ(5,10) (NAME(I),I=1,8)
       READ(5,11)NORIVA,NFREEVA,NKEQ,NKINEQ,IYOFX,ICTYPE
    11 FORMAT(6I5)
       NKTOT=NKEQ+NKINEQ
       READ(5,12)(IFREE(I),I=1,NORIVA)
    12 FORMAT(40I2)
       READ(5,13)TOLCON ,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLY,TOLKUN
    13 FORMAT(8F10.6)
       READ(5,11)MAXITER,MAXLEV,        ICON
       IF(ICON.LT.0) CALL DATAOUT(ITITLE,ISUBTIT,NAME,IYOFX,ICTYPE,
      1NFREEVA)


       Provide an initial feasible solution, X(i), here.















       CALL DIFALGO(1)
   124 WRITE(6,133)            (X(J),J=1,NC),Y
   133 FORMAT(1H ,15F7.0,1X,E11.3)
   115 CONTINUE
       STOP
       END
```

CODE LISTING OF CONTROL

```
      SUBROUTINE CONTROL(I,J,K,L)
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ T0LCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      NC=NC1(1)
C
C
C        THIS SUBROUTINE IS THE USER SUPPLIED OBJECTIVE FUNCTION,
C        CONSTRAINTS, AND ASSOCIATED DERIVATIVES WITH RESPECT TO THE
C        INDEPENDENT VARIABLES
C
C
C
C
C        DEFINITION 0F THE OBJECTIVE FUNCTION
C
C
      IF(I .NE.1) GO TO 1

      List statements necessary to define the value of the objective
      function,Y, here.

      IF(IPRINT.GT.0) GO TO 1
      WRITE(6,102) Y
  102 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1CONTROL*/11X,*VALUE OF THE FUNCTION IS *,F20.4)
C
C
C        DEFINITION 0F PROBLEM CONSTRAINTS--MUST BE WRITTEN IN
C        GREATER-THAN-OR-EQUAL-TO FORMAT
C
C
    1 IF(J .NE.2) GO TO 2

      List statements necessary to define each constraint here.

      Constraints have the form:
          AF(1)= f(x)







      IF(IPRINT.GT.0) GO TO 2
      WRITE(6,103)
  103 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1CONTROL*/11X,*VALUES OF PROBLEM CONSTRAINTS*/11X,* I *,10X,* AF(I)
     2 * )
      DO 10 M=1,NKTOT
   10 WRITE(6,104) M,AF(M)
  104 FORMAT(1H ,11X,I3,10X,E11.4)
C
C
C        DEFINITION OF OBJECTIVE FUNCTION DERIVATIVES
C
C
    2 IF(K.NE.3) GO TO 3
      List statements of the derivative of the objective function,
      ADYDX, here.  One derivative for each variable.


C
C
C        DEFINITION OF CONSTRAINT DERIVATIVES
C
C
    3 IF(L.NE.4) GO TO 4


      List the derivatives of each constraint with respect to each
      variable, ADFDX, here.  This is a double subscript array.



    4 RETURN
      END
```

CODE LISTING OF DIFALGO

```
      SUBROUTINE DIFALGO(NNP)
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),NT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      JOBCODE=NNP
      DO 4 I=1,30
    4 W(I)=0.0
      ID=MAXLEV
      ICODE=10
      APT=10000.0
      IMIN=1
      ICOUNT=0
 1000 CONTINUE
      ICOUNT=ICOUNT+1
      IPRINT=4
      IF(ICOUNT.EQ.ID) IPRINT=-1
      IF(ICOUNT.EQ.ID) ID=ID+MAXLEV
      IF(IPRINT.GT.0) GO TO 1001
      WRITE(6,101) ICOUNT
  101 FORMAT(1H1,10X,60H************************************************
     1************  /11X,*DEBUGGING OUTPUT FOR ITERATION NO. *,I3)
 1001 CONTINUE
      I=1
      J=2
      K=3
      L=4
      CALL CONTROL(I,J,K,L)
      NLK=0
      NTK=0
      N=NKEQ+1
      IF(NKINEQ.EQ.0) GO TO 5
      DO 2 I=N,NKTOT
      IF(ABS(AF(I)).GT.TOLCON) NLK=NLK+1
      IF(ABS(AF(I)).LE.TOLCON) NTK=NTK+1
    2 CONTINUE
    5 CONTINUE
      NSV=NTK+NKEQ
      NDV=NORIVA-NKEQ
      IF(ICOUNT.EQ.1) CALL ARRAY
      CALL REORGA
      IF(IREG.LT.0) WRITE(6,3)
    3 FORMAT(1H ,10X,*THE JACOBIAN MATRIX IS SINGULAR*)
      IF(IREG.LT.0) GO TO 473
      I1=1
      I2=I3=I4=I5=I6=2
      CALL KODRIV(I1,I2,I3,I4,I5,I6)
      CALL KUNTUK
      IF(IMIN.EQ.10) GO TO 473
      P1=ABS(APT)-ABS(Y)
      IF(ABS(P1).LE.TOLY) IMIN=3
      IF(ABS(P1).LE.TOLY) CALL ANSOUT
      IF(IMIN.EQ.10) GO TO 473
      APT=Y
      CALL JORK
      IF(ICODE.EQ.1) CALL DECDJ
      IF(ICODE.EQ.2) CALL INCDJ
      IF(ICODE.EQ.3) CALL INCFT
      IF(IMIN.EQ.10) GO TO 473
      IF(ICOUNT.LT.MAXITER) GO TO 1000
      IMIN=2
      CALL ANSOUT
  473 CONTINUE
      RETURN
      END
```

CODE LISTING OF REORGA

```
      SUBROUTINE REORGA
      COMMON /CONT/ NORIVA,NNINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),NT(30),KL(30),IFNEE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTOD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      KLP=0
   41 CONTINUE
      IF(NSV.EQ.0) GO TO 80
      DO 10 I=1,NSV
      IF(I.GT.NORIVA) ADYDS(I)=0.0
      IF(I.GT.NORIVA) GO TO 10
      NAUX=NS(I)
      ADYDS(I)=ADYDX(NAUX)
   10 CONTINUE
   80 M=NORIVA-NSV
      DO 20 I=1,NDV
      IF(NSV.GT.NORIVA) ADYDD(I)=0.0
      IF(NSV.GT.NORIVA) GO TO 20
      IF(I.GT.M) ADYDD(I)=0.0
      IF(I.GT.M) GO TO 20
      NAUX=ND(I)
      ADYDD(I)=ADYDX(NAUX)
   20 CONTINUE
      IF(NSV.EQ.0) GO TO 81
      DO 40 K=1,NSV
      KA=KT(K)
      DO 30 I=1,NSV
      IF(I.GT.NORIVA.AND.I.EQ.K) ADFTDS(K,I)=-1.0
      IF(I.GT.NORIVA.AND.I.EQ.K) GO TO 30
      IF(I.GT.NORIVA.AND.I.NE.K) ADFTDS(K,I)=0.0
      IF(I.GT.NORIVA.AND.I.NE.K) GO TO 30
      NA=NS(I)
      ADFTDS(K,I)=ADFDX(KA,NA)
   30 CONTINUE
      DO 35 J=1,NDV
      IF(J.GT.M.AND.M.EQ.J) ADFTDD(K,J)= 1.0
      IF(J.GT.M.AND.J.EQ.K) GO TO 35
      IF(J.GT.M.AND.J.NE.K) ADFTDD(K,J)=0.0
      IF(J.GT.M.AND.J.NE.K) GO TO 35
      NA=ND(J)
      ADFTDD(K,J)=ADFDX(KA,NA)
   35 CONTINUE
   40 CONTINUE
   81 IF(NLK.EQ.0) GO TO 63
      DO 60 L=1,NLK
      LA=KL(L)
      IF(NSV.EQ.0) GO TO 82
      DO 50 I=1,NSV
      IF(I.GT.NORIVA.AND.I.EQ.L) ADFLDS(L,I)=-1.0
      IF(I.GT.NORIVA.AND.I.EQ.L) GO TO 50
      IF(I.GT.NORIVA.AND.I.NE.L) ADFLDS(L,I)=0.0
      IF(I.GT.NORIVA.AND.I.NE.L) GO TO 50
      NA=NS(I)
      ADFLDS(L,I)=ADFDX(LA,NA)
   50 CONTINUE
   52 DO 55 J=1,NDV
      IF(M.LE.0.AND.J.EQ.L) ADFLDD(L,J)=1.0
      IF(M.LE.0.AND.J.EQ.L) GO TO 55
      IF(M.LE.0.AND.J.NE.L) ADFLDD(L,J)=0.0
      IF(M.LE.0.AND.J.NE.L) GO TO 55
      IF(J.GT.M.AND.J.EQ.L) ADFLDD(L,J)=1.0
      IF(J.GT.M.AND.J.EQ.L) GO TO 55
      IF(J.GT.M.AND.J.NE.L) ADFLDD(L,J)=0.0
      IF(J.GT.M.AND.J.NE.L) GO TO 55
      NA=ND(J)
      ADFLDD(L,J)=ADFDX(LA,NA)
   55 CONTINUE
   60 CONTINUE
   63 IF(NSV.EQ.0) IREG=2
      IF(NSV.EQ.0) GO TO 61
      CALL JACOBI
      IF(IREG.GE.0) GO TO 61
      KLP=KLP+1
      NP=NSV+1
      IF(KLP.GT.NP) GO TO 61
      IF(KLP.EQ.1) N1=1
      IF(KLP.EQ.1) N2=NSV
      IF(KLP.GT.1) N1=KLP-1
      IF(KLP.GT.1) N2=N1
      DO 42 I=N1,N2
      NAUX=KT(I)
      NAUXT=NS(I)
      XTY=0.0
      MPT=0
      IF(KLP.EQ.1.AND.ABS(ADFDX(NAUX,NAUXT)).GT.TOLVJ) GO TO 42
      DO 43 J=1,NDV
      IF(J.GT.M) GO TO 43
      NAU=ND(J)
      IF(ABS(X(NAU)).LE.TOLDJ) GO TO 43
      IF(ABS(ADFDX(NAUX,NAU)).GT.TOLVJ) GO TO 44
      GO TO 43
   44 IF(ABS(X(NAU)).GT.XTY) NA=NAU
      IF(ABS(X(NAU)).GT.XTY) K=J
      IF(ABS(X(NAU)).GT.XTY) XTY=X(NAU)
      MPT=1
   43 CONTINUE
      IF(MPT.EQ.0) GO TO 42
      NS(I)=NA
      ND(K)=NAUXT
   42 CONTINUE
      GO TO 41
   61 CONTINUE
      IF(IPRINT.GT.0) GO TO 101
      WRITE(6,102)
  102 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1REORGA*//11X,*I*,5X,*NS(I)*,*     S     *,10X,*ND(I)*,*     D     *
     1)
      DO 104 I=1,NORIVA
      IF(I.GT.NSV.AND.I.GT.M) GO TO 104
      IF(I.GT.NSV) WRITE(6,107) I
  107 FORMAT(1H ,8X,I3)
      IF(I.GT.NSV) GO TO 105

      NAUX=NS(I)
      WRITE(6,103) I,NAUX,X(NAUX)
  103 FORMAT(1H ,8X,I3,4X,I5,F10.3)
  105 CONTINUE
      IF(I.GT.M) GO TO 104
      NAU =ND(I)
      WRITE(6,106)   NAU ,X(NAU)
  106 FORMAT(1H+,40X,I5,F10.3)
  104 CONTINUE
  101 RETURN
      END
```

CODE LISTING OF ARRAY

```
      SUBROUTINE ARRAY
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      I=J=K=L=M=0
      IF(NSV.EQ.0) GO TO 3
    5 I=I+1
    4 K=K+1
      IF(K.GT.NORIVA) GO TO 2
      IF(IFREE(K).GT.0) GO TO 4
      NS(I)=K
      IF(I.GE.NSV) GO TO 3
      GO TO 5
    2 CONTINUE
      K=0
    7 K=K+1
      IF(K.GT.NORIVA) GO TO 6
      IF(IFREE(K).LT.0) GO TO 7
      KK=K
      IF(X(KK).LE.TOLONS) GO TO 7
      NS(I)=K
      IF(I.GE.NSV) GO TO 3
      I=I+1
      GO TO 7
    6 CONTINUE
      K=0
    8 K=K+1
      IF(K.GT.NORIVA) GO TO 3
      IF(IFREE(K).LT.0) GO TO 8
      IF(X(K).GT.0.0) GO TO 8
      NS(I)=K
      IF(I.GE.NSV) GO TO 3
      I=I+1
      GO TO 8
    3 CONTINUE
      K=0
    9 J=J+1
   10 K=K+1
      IF(K.GT.NORIVA) GO TO 12
      IF(NSV.EQ.0) GO TO 19
      DO 11 M=1,I
      IF(NS(M).EQ.K) GO TO 10
   11 CONTINUE
   19 CONTINUE
      ND(J)=K
      IF(J.GE.NDV) GO TO 12
      GO TO 9
   12 CONTINUE
      IF(NSV.EQ.0) GO TO 15
      K=L=0
   14 L=L+1
   13 K=K+1
```

```
      IF(K.GT.NKTOT) GO TO 15
      IF(ABS(AF(K)).GT.TOLCON) GO TO 13
      KT(L)=K
      GO TO 14
   15 CONTINUE
      L=K=0
   16 L=L+1
   17 K=K+1
      IF(K.GT.NKTOT) GO TO 18
      IF(K.LE.NKEQ) GO TO 17
      IF(ABS(AF(K)).LE.TOLCON) GO TO 17
      KL(L)=K
      GO TO 16
   18 CONTINUE
      RETURN
      END
      SUBROUTINE JAC0BI
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ T0LCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),RT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      NDESIZ=NTK+NKEQ
      IREG=0
      DO 12 K=1,NSV
      DO 12 I=1,NSV
      AMA(K,I)=ADFTDS(K,I)
   12 CONTINUE
      IF(NDESIZ.EQ.1) DETERM=AMA(1,1)
      IF(NDESIZ.EQ.1) GO TO 13
      CALL GAUSS
   13 CONTINUE
      AJACOBI=DETERM
      IF(ABS(AJACOBI).LE.TOLJAC) IREG=-1
      IF(IPRINT.GT.0) GO TO 101
      WRITE(6,102) AJACOBI
  102 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1JACOBI*/11X,*VALUE OF JACOBIAN MATRIX DETERMINANT IS*/11X,F20,3)
  101 RETURN
      END
```

CODE LISTING FOR JORK

```
      SUBROUTINE JORK
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),NT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      JPARP=0
      KPARN=0
      AVP=0.0
      AVN=-0.0
      AVTOT=0.0
      M=NORIVA-NSV
      DO 9 J=1,NDV
      IF(J.GT.M) D=0.0
      IF(J.GT.M) GO TO 9
      NAUX=ND(J)
      D=X(NAUX)
   30 CONTINUE
      C=CODYDD(J)
      IF(D.GT.TOLONS.AND.C.GT.AVP) JPARP=J
      IF(D.GT.TOLONS.AND.C.GT.AVP) AVP=C
      IF(C.LT.AVN) JPARN=J
      IF(C.LT.AVN) AVN=C
    9 CONTINUE
      IF(NSV.EQ.NKEQ) GO TO 18
      IF(NSV.EQ.0) GO TO 18
      KPARN=0
      JERK=NKEQ+1
      DO 19 K=JERK,NSV
      C=CODYDF(K)
      IF(C.LE.AVN) KPARN=K
      IF(C.LE.AVN) AVN=C
   19 CONTINUE
   18 CONTINUE
   17 AVTOT=AVP+AVN
      IF(AVTOT.GE.0.0) GO TO 20
      KPAR=KPARN
      IF(KPARN.EQ.0) GO TO 21
      ICODE =3
      GO TO 22
   20 JPAR=JPARP
      ICODE=1
      GO TO 22
   21 JPAR=JPARN
      ICODE=2
   22 CONTINUE
      IF(IPRINT.GT.0) GO TO 101
      WRITE(6,102) ICODE,JPAR,KPARN
      IF(JPAR.EQ.0.AND.KPAR.EQ.0) ICODE=10
      IF(JPAR.EQ.0.AND.KPAR.EQ.0) IMIN=1
  102 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1JORK*/11X,*ICODE*,5X,*JPAR *,5X,*KPAR */11X,I5,5X,I5,5X,I5)
  101 RETURN
      END
```

CODE LISTING FOR DECDJ

```
      SUBROUTINE DECDJ
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREQ,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFDS(3
     10,30),ADFDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      COMMON X1(30)
      REAL MAXDSI,MAXDFL,MAXDD
      JVP=JPAR
      NAUX=ND(JVP)
      VALXO=X(NAUX)
      DO 201 I=1,NORIVA
  201 X1(I)=X(I)
      VALYO=CODYDD(JVP)
  101 CONTINUE
      KLW=0
      KPARP=-1
      IPARP=-1
      APPLE=-80000.0
      IF(NSV.NE.0) CALL KODRIV(3,3,3,3,3,3)
      IF(NLK.NE.0) CALL KODRIV(4,4,4,4,4,4)
  103 ASVP=APPLE
      IF(NSV.EQ.0) GO TO 32
      DO 20 I=1,NSV
      IF(KLW.EQ.IPARP.AND.I.EQ.IPARP) GO TO 20
      NAUX=NS(I)
      C=CODSDD(I,JVP)
      IF(IFREE(NAUX).LT.0) GO TO 20
      IF(C.LE.0.0) GO TO 20
      MAXDSI=-X(NAUX)/C
      IF(MAXDSI.GT.ASVP) IPARP=I
      IF(MAXDSI.GT.ASVP) ASVP=MAXDSI
   20 CONTINUE
   32 ASFL=APPLE
      IF(NLK.EQ.0) GO TO 31
      DO 30 I=1,NLK
      IF(KLW.EQ.KPARP.AND.I.EQ.KPARP) GO TO 30
      NAUX=KL(I)
      C=CODFLDD(I,JVP)
      IF(C.LE.0.0) GO TO 30
      MAXDFL=-AF(NAUX)/C
      IF(MAXDFL.GT.ASFL) KPARP=I
      IF(MAXDFL.GT.ASFL) ASFL=MAXDFL
   30 CONTINUE
   31 CONTINUE
      NAUX=ND(JVP)
      MAXDD=-ABS(X(NAUX))
      IF(IFREE(NAUX).LT.0) MAXDD=APPLE
      IF(ASVP.GE.0.0) KLW=IPARP
      IF(ASFL.GE.0.0) KLW=KPARP
      IF(KLW.EQ.IPARP) GO TO 103
      IF(KLW.EQ.KPARP) GO TO 103
      MAXDSI=ASVP
      MAXDFL=ASFL
      APT=ABS(MAADSI-MAXDFL)+ABS(MAXDSI-MAXDD)+ABS(MAXDFL-MAXDD)
      APC=3.0*TOLCON
      IF(APT.LE.APC)  MAXDD=MAXDD/2.0
      IF(APT.LE.APC)   GO TO 50
      IF(MAXDSI.GT.MAXDFL) GO TO 71
      IF(MAXDFL.GT.MAXDD) GO TO 45
      GO TO 50
   71 IF(MAXDSI.GT.MAXDD) GO TO 40
      GO TO 50
   40 NAUX=ND(JVP)
      X(NAUX)=X(NAUX7+MAXDSI
      NAUX=NS(IPARP)
      X(NAUX)=0.0
      DO 41 I=1,NSV
      IF(I.EQ.IPARP) GO TO 41
      NAUX=NS(I)
      X(NAUX)=X(NAUX7+MAXDSI*CODSDD(I,JVP)
   41 CONTINUE
      CALL CONTROL(2,2,3,4)
      CALL REOHGA
      CALL KODRIV(1,1,1,1,1,1)
      IF(CODYDD(JVP).LT.0.0) GO TO 60
      NAUX=ND(JVP)
      NAU=NS(IPARP)
      NS(IPARP)=NAUX
      ND(JVP)=NAU
      CALL NEWTSIM(17
      IF(IMIN.EQ.10) GO TO 102
      CALL ENDCHEK(17
      GO TO 102
   45 NAUX=ND(JVP)
      X(NAUX)=X(NAUX7+MAXDFL
      IF(NSV.EQ.0) GO TO 47
      DO 46 I=1,NSV
      NAUX=NS(I)
      X(NAUX)=X(NAUX7+MAXDFL*CODSDD(I,JVP)
   46 CONTINUE
   47 CALL CONTROL(2,2,3,4)
      CALL REOHGA
      CALL KODRIV(1,1,1,1,1,1)
      IF(CODYDD(JVP).LT.0.0) GO TO 60
      XMAX=0.0
      NZ=NORIVA-NSV
      DO 48 I=1,NDV
      IF(I.GT.NZ) GO TO 48
      NAUX=ND(I)
      IF(X(NAUX).LT. XMAX) GO TO 48
      XMAX=X(NAUX)
      JCH=I
   48 CONTINUE
      NSV=NSV+1
      NTK=NTK+1
      NS(NSV)=ND(JCH)
      KT(NSV)=KL(KPARP)
      ND(JCH)=ND(NZ)
      KL(KPARP)=KL(NLK)
      NLK=NLK-1
      CALL REOHGA
      M=NORIVA-NSV
      IF(JPAR.GE.M)  JPAR=M
      IF(M.LE.0) GO TO 102
      CALL NEWTSIM(1)
      IF(IMIN.EQ.10) GO TO 102
      CALL ENDCHEK(1)
      GO TO 102
   50 NAUX=ND(JVP)
      D=ABS(5.0*X(NAUX))
      IF(D.LT.ABS(MAXDD)) MAXDD=-D
      IF(D.LT.ABS(MAXDD)) GO TO 53
      X(NAUX)=X(NAUX)+MAXDD
   53 CONTINUE
      IF(NSV.EQ.0) GO TO 52
      DO 51 I=1,NSV
      NAUX=NS(I)
      X(NAUX)=X(NAUX)+MAXDD*CODSDD(I,JVP)
   51 CONTINUE
   52 CALL CONTROL(2,2,3,4)
      CALL REORGA
      CALL KODMIV(1,1,1,1,1,1)
      IF(CODYDD(JVP).LT.0.0) GO TO 60
      CALL NEWTSIM(1)
      IF(IMIN.EQ.10) GO TO 102
      CALL ENDCHEK(1)
      GO TO 102
   60 CONTINUE
      ICO=0
      ICP=0
      VALYN=CODYDD(JVP)
      NAUX=ND(JVP)
      VALXN=X(NAUX)
   61 CONTINUE
      ICP=ICP+1
      ICO=ICO+1
      IF(ICO.GT.30) GO TO 102
      IF(ICP.LE.3) XZERO=VALXN+(VALXO-VALXN)/2.0
      IF(ICP.GT.3) XZERO=(ABS(VALYN)*VALXO+ABS(VALYO)*VALXN)/(ABS(VALYN)
     1+ABS(VALYO))
      NAUX=ND(JVP)
      IF(ICO.EQ.1) GO TO  906
      X1(NAUX)=X(NAUX)
  906 X(NAUX)=XZERO
      CALL NEWTSIM(1)
      IF(IMIN.EQ.10) GO TO 102
      XZERO=X(NAUX)
      CALL CONTROL(2,2,3,4)
      CALL REORGA
      CALL KODMIV(1,1,1,1,1,1)
      IF(ICODE.LE.0.AND.ICO.GT.15) GO TO 102
      IF(ABS(CODYDD(JVP)).LE.TOLVJ) GO TO 100
      IF(CODYDD(JVP).GT.TOLVJ) VALXO=XZERO
      IF(CODYDD(JVP).GT.TOLVJ) VALYO=CODYDD(JVP)
      IF(CODYDD(JVP).GT.TOLVJ) GO TO 61
      IF(CODYDD(JVP).LT.TOLVJ) VALXN=XZERO
      IF(CODYDD(JVP).LT.TOLVJ) VALYN=CODYDD(JVP)
      IF(CODYDD(JVP).LT.TOLVJ) GO TO 61
  100 CONTINUE
      CALL ENDCHEK(1)


  102 CONTINUE
      IF(IPRINT.GT.0) GO TO 104
      WRITE(6,909)
  909 FORMAT(1H ,10H********** ,*I AM HERE AT DECOJ*,10H********** )
  104 RETURN
      END
```

CODE LISTING OF INCDJ

```
      SUBROUTINE INCDJ
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      COMMON X1(30)
      REAL MINDSI,MINDFL
      JVN=JPAR
      NAUX=ND(JVN)
      VALXO=X(NAUX)
      DO 201 I=1,NORIVA
  201 X1(I)=X(I)
      VALYO=CODYDD(JVN)
  101 CONTINUE
      KLW=0
      KPARP=-1
      IPARP=-1
      APPLE=10000.0
      IF(NSV.NE.0) CALL KODRIV(3,3,3,3,3)
      IF(NLK.NE.0) CALL KODRIV(4,4,4,4,4)
   17 ADSI=APPLE
      IF(NSV.EQ.0) GO TO 22
      DO 20 I=1,NSV
      IF(KLW.EQ.IPARP.AND.I.EQ.IPARP) GO TO 20
      NAUX=NS(I)
      IF(IFREE(NAUX).LT.0) GO TO 20
      C=CODSDD(I,JVN)
      IF(C.GE.0.0) GO TO 20
      IF(ABS(C).LE.TOLVJ) GO TO 20
      MINDSI=-X(NAUX)/C
      IF(MINDSI.LT.ADSI) IPARP=I
      IF(MINDSI.LT.ADSI) ADSI=MINDSI
   20 CONTINUE
   22 ADFL=APPLE
      IF(NLK.EQ.0) GO TO 31
      DO 30 I=1,NLK
      IF(KLW.EQ.KPARP.AND.I.EQ.KPARP) GO TO 30
      NAUX=KL(I)
      C=CODFLDD(I    ,JVN)
      IF(C.GE.0.0) GO TO 30
      IF(ABS(C).LE.TOLVJ) GO TO 30
      MINDFL=-AF(NAUX)/C
      IF(MINDFL.LT.ADFL) KPARP=I
      IF(MINDFL.LT.ADFL) ADFL=MINDFL
   30 CONTINUE
   31 CONTINUE
      IF(ADSI.LE.0.0) KLW=IPARP
      IF(ADFL.LE.0.0) KLW=KPARP
      IF(KLW.EQ.IPARP) GO TO 17
      IF(KLW.EQ.KPARP) GO TO 17
      MINDFL=ADFL
      MINDSI=ADSI
      IF(MINDFL.NE.APPLE) GO TO 21
      IF(MINDSI.NE.APPLE) GO TO 21
      NAUX=ND(JVN)
      MINDSI=3.0*ABS(X(NAUX))+1.0
   21 CONTINUE
      IF(MINDSI.LT.MINDFL) GO TO 150
      GO TO 160
  150 CONTINUE
      IF(IPARP.LE.0) GO TO 152
      NAUX=NS(IPARP)
      X(NAUX)=0.0
  152 CONTINUE
      NAUX=ND(JVN)
      X(NAUX)=X(NAUX)+MINDSI
      IF(NSV.EQ.0) GO TO 153
      DO 41 I=1,NSV
      IF(I.EQ.IPARP) GO TO 41
      NAUX=NS(I)
      X(NAUX)=X(NAUX)+MINDSI*CODSDD(I,JVN)
   41 CONTINUE
  153 CALL CONTROL(2,2,3,4)
      CALL REOHGA
      CALL KODMIV(1,1,1,1,1,1)
      IF(CODYDD(JVN).GT.0.0) GO TO 1100
      IF(NSV.EQ.0) GO TO 100
      IF(IPARP.LE.0) GO TO 154
      NAUX=ND(JVN)
      NAU=NS(IPARP)
      NS(IPARP)=NAUX
      ND(JVN)=NAU
  154 CONTINUE
      CALL NEWTSIM(2)
      IF(IMIN.EQ.10) GO TO 102
      CALL ENDCHEK(2)
      GO TO 102
  160 CONTINUE
      NAUX=ND(JVN)
      X(NAUX)=X(NAUX)+MINDFL
      IF(NSV.EQ.0) GO TO 23
      DO 46 I=1,NSV
      NAUX=NS(I)
      X(NAUX)=X(NAUX)+MINDFL*CODSDD(I,JVN)
   46 CONTINUE
   23 CALL CONTROL(2,2,3,4)
      CALL REOHGA
      CALL KODMIV(1,1,1,1,1,1)
      IF(CODYDD(JVN).GT.0.0) GO TO 1100
      XMAX=0.0
      N2=NORIVA-NSV
      DO 48 I=1,NDV
      IF(I.GT.N2) GO TO 48
      NAUX=ND(I)
      IF(ABS(X(NAUX)).LT.XMAX) GO TO 48
      XMAX=X(NAUX)
      JCH=I
   48 CONTINUE
      NSV=NSV+1
      NTR=NTR+1
      NS(NSV)=ND(JCH)
      KT(NSV)=KL(KPARP)
      ND(JCH)=ND(NZ)
      KL(KPARP)=KL(NLK)
      NLK=NLK-1
      CALL REOMGA
      M=NORIVA-NSV
      IF(JPAR.GE.M) JPAR=M
      IF(M.LE.0) GO TO 102
      CALL NEWTSIM(2)
      IF(IMIN.EQ.10) GO TO 102
      CALL ENDCHEK(2)
      GO TO 102
 1100 CONTINUE
      ICP=0
      ICO=0
      NAUX=ND(JVN)
      VALXN=X(NAUX)
      VALYN=CODYDD(JVN)
   61 CONTINUE
      ICP=ICP+1
      ICO=ICO+1
      IF(ICO.GT.30) GO TO 104
      IF(ICP.LE.3) XZERO=VALXO+(VALXN-V/XO)/2.0
      IF(ICP.GT.3) XZERO=(ABS(VALYN)*VAKO+ABS(VALYO)*VALXN)/(ABS(VALYN)
     1+ABS(VALYO))
      NAUX=ND(JVN)
      IF(ICO.EQ.1) GO TO  906
      X1(NAUX)=X(NAUX)
  906 X(NAUX)=XZERO
      CALL NEWTSIM(2)
      IF(IMIN.EQ.10) GO TO 102
      CALL CONTROL(2,2,3,4)
      CALL REOMGA
      CALL XODMIV(1,1,1,1,1,1)
      IF(ICODE.LE.0.AND.ICO.GT.15) GO T 102
      IF(ABS(CODYDD(JVN)).LE.TOLVJ) GO 0 100
      IF(CODYDD(JVN).GT.TOLVJ) VALXN=XIRO
      IF(CODYDD(JVN).GT.TOLVJ) VALYN=C)YDD(JVN)
      IF(CODYDD(JVN).GT.TOLVJ) GO TO 61
      IF(CODYDD(JVN).LT.TOLVJ) VALXO=XIRO
      IF(CODYDD(JVN).LT.TOLVJ) VALYO=C)YDD(JVN)
      IF(CODYDD(JVN).LT.TOLVJ) GO TO 61
  100 CONTINUE
      CALL ENDCHEK(2)
  102 CONTINUE
      IF(IPRINT.GT.0) GO TO 104
      WRITE(6,931)
  931 FORMAT(1H ,10X,*I AM HERE AT END*)
  104 RETURN
      END
```

CODE LISTING OF INCFT

```
      SUBROUTINE INCFT
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      COMMON X1(30)
      REAL MINDFS,MINDFL
      JFT=KPAR
      DO 201 I=1,NORIVA
  201 X1(I)=X(I)
      APPLE=10000.0
      CALL  KODRIV(5,5,5,5,5,5)
      IF( NLK.EQ.0) GO TO 99
      CALL KODRIV(6,6,6,6,6,6)
   99 CONTINUE
      MINDFS=APPLE
      MINDFL=APPLE
      KLW=0
      IFS=-1
      IFL=-1
      DO 100 I=1,NSV
      IF(KLW.EQ.IFS.AND.I.EQ.IFS) GO TO 100
      NAUX=NS(I)
      IF(IFREE(NAUX).LT.0) GO TO 100
      IF(CODSDF(I,JFT).GE.0.0) GO TO 100
      ADU=-X(NAUX)/CODSDF(I,JFT)
      IF(ADU.GE.MINDFS) GO TO 100
      MINDFS=ADU
      IFS=I
  100 CONTINUE
      IF(NLK.EQ.0) GO TO 102
      DO 101 I=1,NLK
      IF(KLW.EQ.IFL.AND.I.EQ.IFL) GO TO 101
      NAUX=KL(I)
      IF(CODFLDF(I,JFT).GE.0.0) GO TO 101
      ADU=-AF(NAUX)/CODFLDF(I,JFT)
      IF(ADU.GE.MINDFL) GO TO 101
      MINDFL=ADU
      IFL=I
  101 CONTINUE
  102 CONTINUE
      IF(MINDFL.LE.0.0) KLW=IFL
      IF(MINDFS.LE.0.0) KLW=IFS
      IF(KLW.EQ.IFL) GO TO 97
      IF(KLW.EQ.IFS) GO TO 98
      TEMP=ABS(MINDFL)-ABS(MINDFS)
      TEM=3.0*TOLONS
      IF(ABS(TEMP).LE.TEM) MINDFL=0.45
      IF(MINDFL.GT.MINDFS) GO TO 500
      DO 111 I=1,NSV
      NAUX=NS(I)
  111 X(NAUX)=X(NAUX)-MINDFL*CODSDF(I,JFT)*0.75
```

```
      AP=APPLE
      N=0
      DO 600 I=1,NSV
      NAUX=NS(I)
      IF(ABS(X(NAUX)).LE.AP) N=I
      IF(ABS(X(NAUX)).LE.AP) AP=X(NAUX)
  600 CONTINUE
      IFS=N
      GO TO 601
  500 CONTINUE
      DO 511 I=1,NSV
      NAUXT=NS(I)
      X(NAUXT)=X(NAUXT)+CODSDF(I   ,JFT)*WINDFS*0.75
  511 CONTINUE
  601 CONTINUE
      ID=0
      NAUX=NS(IFS)
      DO 512 I=1,NSV
      IF(I.EQ.IFS) GO TO 512
      ID=ID+1
      NS(ID)=NS(I)
  512 CONTINUE
      NSV=NSV-1
      NZ=NORIVA-NSV
      NLK=NLK+1
      ID=0
      DO 513 I=1,NDV
      IF(I.LT.NZ) GO TO 513
      IF(I.GT.NZ) GO TO 513
      ND(I)=NAUX
  513 CONTINUE
      KL(NLK)=KT(JFT)
      NTK=NTK-1
      IF(NSV.EQ.NKEQ) GO TO 515
      ID=NKEQ
      N1=NKEQ+1
      DO 514 I=N1,NSV
      IF(I.EQ.JFT) GO TO 514
      ID=ID+1
      KT(ID)=KT(I)
  514 CONTINUE
  515 CONTINUE
      CALL NEWISIM(3)
      IF(IMIN.EQ.10) GO TO 37
      CALL ENDCHEK(3)
   37 CONTINUE
      IF(IPRINT.GT.0) GO TO 104
      WRITE(6,301)
  301 FORMAT(1H ,*I AM HERE AT SUBROUTINE INCFT*)
  104 RETURN
      END
```

CODE LISTING OF KODRIV

```
      SUBROUTINE KODRIV(I1,I2,I3,I4,I5,I6)
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTR,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NCI(30),
     1ND(30),NS(30),NT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      IF(I1.NE.1) GO TO 301
      NDESIZ=NTK+NKEQ+1
      IREG=0
      M=NORIVA-NSV
      IF(M.LE.0) GO TO 301
      DO 9 J=1,M
      AMA(1,1)=ADYDD(J)
      IF(NDESIZ.EQ.1) CODYDD(J)=AMA(1,1)
      IF(NDESIZ.EQ.1) GO TO 9
      DO 2 JC=2,NDESIZ
      I=JC-1
      AMA(1,JC)=ADYDS(I)
    2 CONTINUE
      DO 3 IR=2,NDESIZ
      K=IR-1
      AMA(IR,1)=ADFTDD(K,J)
    3 CONTINUE
      DO 5 IR=2,NDESIZ
      DO 4 JC=2,NDESIZ
      K=IR-1
      I=JC-1
      AMA(IR,JC)=ADFTDS(K,I)
    4 CONTINUE
    5 CONTINUE
      CALL GAUSS
    6 CONTINUE
      CODYDD(J)=DETERM/AJACOBI
    9 CONTINUE
      IF(IPRINT.GT.0) GO TO 301
      WRITE(6,102)
  102 FORMAT(1H0,10X,30H******************************  /11X,*SUBROUTINE
     1KODRIV*/11X,* I *,10X,* CODYDD(I)*)
      DO 103 I=1,M
      WRITE(6,104) I,CODYDD(I)
  104 FORMAT(1H ,10X,I3,10X,F20.4)
  103 CONTINUE
  301 IF(I2.NE.2) GO TO 302
      NDESIZ=NTK+NKEQ
      IREG=0
      IF(NDESIZ.EQ.1) GO TO 10
      IF(NSV.EQ.0) GO TO 302
      DO 1 I=1,NSV
      AMA(1,I)=ADYDS(I)
    1 CONTINUE
      DO 90 J=1,NSV
      [illegible]
      DO 8 IR=2,NSV
      DO 7 I=1,NSV
      IF(IR.LE.K) IA=IR-1
      IF(IR.GT.K) IA=IR
      AMA(IR,I)=ADFTDS(IA,I)
    7 CONTINUE
    8 CONTINUE
      CALL GAUSS
      CODYDF(K)=(DETERM/AJACOBI)*(-1)**IX
   92 CONTINUE
      IF(NDESIZ.GT.1) GO TO 15
   10 CODYDF(1)=ADYDS(1)/AJACOBI
   15 CONTINUE
      IF(IPRINT.GT.0) GO TO 302
      WRITE(6,202)
  202 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1KODRIV*/11X,* I *,10X,* CODYDF(I)*)
      DO 203 I=1,NSV
      WRITE(6,204) I,CODYDF(I)
  204 FORMAT(1H ,10X,I3,10X,F20.4)
  203 CONTINUE
  302 IF(I3.NE.3) GO TO 303
      JVP=JPAR
      NDESIZ=NSV
      DO 50 M=1,NSV
      DO 20 J=1,NSV
      DO 30 K=1,NSV
      IF(J.EQ.M) GO TO 40
      AMA(K,J)=ADFTDS(K,J)
   30 CONTINUE
      GO TO 20
   40 DO 45 K=1,NSV
      AMA(K,J)=ADFTDD(K,JVP)
   45 CONTINUE
   20 CONTINUE
      IF(NDESIZ.EQ.1) DETERM=AMA(1,1)
      IF(NDESIZ.EQ.1) GO TO 21
      CALL GAUSS
   21 CONTINUE
      CODSDD(M,JVP)=-DETERM/AJACOBI
   50 CONTINUE
      IF(IPRINT.GT.0) GO TO 303
      WRITE(6,12)
   12 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1KODRIV*/13X,*M*,5X,*JVP*,5X,*CODSDD(M,JVP)*)
      WRITE(6,22) (M,JVP,CODSDD(M,JVP),M=1,NSV)
   22 FORMAT(1H ,10X,I3,6X,I3,6X,F20.3)
  303 IF(I4.NE.4) GO TO 304
      JVP=JPAR
      IF(NLK.EQ.0) GO TO 304
      DO 26 I=1,NLK
      NAUX=KL(I)
      IF(NSV.EQ.0) CODFLDD(I,JVP)=ADFDS(NAUX,JVP)
      IF(NSV.EQ.0) GO TO 26
      AMA(1,1)=ADFLDD(I,JVP)
      DO 26 J=1,NSV
      JC=J+1
      AMA(1,JC)=ADFLDS(I,J)
   26 CONTINUE
      DO 28 K=1,NSV
      JK=K+1
      AMA(JR,1)=ADFTDD(K,JVP)
   28 CONTINUE
      DO 32 L=1,NSV
      JC=L+1
      DO 34 M=1,NSV
      JR=M+1
      AMA(JR,JC)=ADFTDS(M,L)
   34 CONTINUE
   32 CONTINUE
      NDESIZ=NSV+1
      CALL GAUSS
      CODFLDD(I,JVP)=DETERM/AJACOBI
   24 CONTINUE
      IF(IPRINT.GT.0) GO TO 304
      WRITE(6,48)
   48 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1KODRIV */13X,*I*,5X,*CODFLDD(I,JVP)*)
      WRITE(6,401) (I,CODFLDD(I,JVP),I=1,NLK)
  401 FORMAT(1H ,10X,I3,5X,F20.3)
  304 IF(I5.NE.5) GO TO 305
      JFT=KPAR
      DO55 N=1,NSV
      IX=JFT+N
      DO520 J=1,NSV
      IF(J.EQ.JFT) GO TO520
      IF(J.LT.JFT) JD=J
      IF(J.GT.JFT) JD=J-1
      DO530 K=1,NSV
      IF(K.EQ.N) GO TO530
      IF(K.GT.N) KD=K-1
      IF(K.LT.N) KD=K
      AMA(JD,KD)=ADFTDS(J,K)
  530 CONTINUE
  520 CONTINUE
      NDESIZ=NSV-1
      IF(NDESIZ.EQ.0) DETERM=1.0
      IF(NDESIZ.EQ.1) DETERM=AMA(1,1)
      IF(NDESIZ.LE.1) CODSDF(N,JFT)=(DETERM/AJACOBI)*(-1)**IX
      IF(NDESIZ.LE.1) GO TO510
      CALL GAUSS
      CODSDF(N,JFT)=(DETERM/AJACOBI)*(-1)**IX
  510 CONTINUE
   55 CONTINUE
      IF(IPRINT.GT.0) GO TO 305
      WRITE(6,502)
  502 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1KODRIV*/11X,* I *,*JFT*,4X,*CODSDF(I,JFT)*)
      DO 504 I=1,NSV
      WRITE(6,505) I,JFT,CODSDF(I,JFT)
  505 FORMAT(1H ,9X,I3,I4,5X,F20.3)
  504 CONTINUE
  305 IF(I6.NE.6) GO TO 306
      JFT=KPAR
      IX=JFT+1
      DO 610 K=1,NLK
      DO 600 I=1,NSV
      ID=I
      IF(I.EQ.JFT) ID=I+1
      DO 600 J=1,NSV
      IF(I.EQ.NSV) GO TO 650
      AMA(I,J)=ADFTDS(ID,J)
      GO TO 600
  650 ID=K
      AMA(I,J)=ADFLDS(ID,J)
  600 CONTINUE
      NDESIZ=NSV
      IF(NDESIZ.EQ.1) CODFLDF(K,JFT)=(AMA(1,1)/AJACOBI)*(-1)**IX
      IF(NDESIZ.EQ.1) GO TO 610
      CALL GAUSS
      CODFLDF(K,JFT)=(DETERM/AJACOBI)*(-1)**IX
  610 CONTINUE
      IF(IPRINT.GT.0) GO TO 306
      WRITE(6,603)
  603 FORMAT(1H0,10X,30H****************************** /11X,*SUBROUTINE
     1KODRIV */11X,* I *,5X,*CODFLDF(I,KPAR)*)
      WRITE(6,602) (I,CODFLDF(I,JFT),I=1,NLK)
  602 FORMAT(1H , 9X,I3,10X,F20.3)
  306 RETURN
      END
```

CODE LISTING OF ENDCHEK

```
      SUBROUTINE ENDCHEK(LCODE)
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAN,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
  407 CONTINUE
      IF(NLK.EQ.0) GO TO 403
  406 AP=TOLCON
      N=0
      CALL CONTROL(2,2,2,2)
      DO 400 I=1,NLK
      NAUX=KL(I)
      IF(AF(NAUX).GT.TOLCON) GO TO 400
      IF(AF(NAUX).LE.AP) N=I
      IF(AF(NAUX).LE.AP) AP=AF(NAUX)
  400 CONTINUE
      IF(N.EQ.0) GO TO 403
      AP=0.0
      M=NORIVA-NSV
      DO 10 I=1,NDV
      IF(I.GT.M) GO TO 10
      NAUX=ND(I)
      IF(X(NAUX).GE.AP)N1=I
      IF(X(NAUX).GE.AP) AP=X(NAUX)
   10 CONTINUE
      NSV=NSV+1
      NTK=NTK-1
      NAUX=KL(N)
      KT(NSV)=NAUX
      KL(N)=KL(NLK)
      NLK=NLK-1
      NAUX=ND(N1)
      NS(NSV)=NAUX
      NAUX=ND(M)
      ND(N1)=NAUX
      CALL REDMBA
      CALL NEWTSIM(LCODE)
  403 CONTINUE
      AP=TOLONS
      N=0
      IF(NSV.EQ.0) GO TO 404
      IF(JOBCODE.LE.0) GO TO 404
      DO 401 I=1,NSV
      NAUX=NS(I)
      IF(IFREE(NAUX).LT.0) GO TO 401
      IF(X(NAUX).GT.TOLONS) GO TO 401
      IF(X(NAUX).LE.AP) N=I
      IF(X(NAUX).LE.AP) AP=X(NAUX)
  401 CONTINUE
      IF(N.EQ.0) GO TO 404
      AP=0.0
      [illegible]
      DO 20 I=1,NDV
      IF(I.GT.M) GO TO 20
      NAUX=ND(I)
      IF(X(NAUX).GE.AP) N1=I
      IF(X(NAUX).GE.AP) AP=X(NAUX)
   20 CONTINUE
      NAUX=NS(N)
      NAUX1=ND(N1)
      NS(N)=NAUX1
      ND(N1)=NAUX
      CALL REDMBA
      GO TO 407
  404 CONTINUE
  405 RETURN
      END
```

CODE LISTING OF KUNTUK

```
      SUBROUTINE KUNTUK
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      IMIN=1
      N1=NORIVA-NSV
      DO 1 J=1,NDV
      IF(J.GT.N1) GO TO 1
      NAUX=ND(J)
      IF(IFREE(NAUX).LT.0) GO TO 1
      IF(X(NAUX).LT.-TOLDJ) IMIN=-1
    1 CONTINUE
      IF(NSV.EQ.0) GO TO 20
      DO 2 I=1,NSV
      K1=KT(I)
      IF(ABS(AF(K1)).GT.TOLCON) IMIN=-2
    2 CONTINUE
   20 DO 3 J=1,NDV
      IF(J.GT.N1) GO TO 3
      IF(CODYDD(J).LT.-TOLVJ) IMIN=-3
    3 CONTINUE
      IF(NSV.EQ.0) GO TO 21
      DO 4 K=1,NSV
      IF(K.LE.NKEQ) GO TO 4
      IF(CODYDF(K ).LT.-TOLVJ) IMIN=-4
    4 CONTINUE
   21 DO 5 J=1,NDV
      IF(J.GT.N1) GO TO 5
      NAUX=ND(J)
      IF(IFREE(NAUX).LT.0) GO TO 6
      TERM=CODYDD(J)*X(NAUX)
      IF(ABS(TERM).GT.TOLKUN) IMIN=-5
      GO TO 5
    6 TERM=CODYDD(J)
      IF(ABS(TERM).GT.TOLKUN) IMIN=-6
    5 CONTINUE
      IF(IMIN)8,8,9
    9 CALL ANSOUT
    8 RETURN
      END
```

CODE LISTING OF NEWTSIM

```
      SUBROUTINE NEWTSIM(JCODE)
      COMMON /CONT/ NORIVA,NRINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      COMMON XI(30)
      ICODE=10
      KP=0
      N=NSV
      IF(N.EQ.0) GO TO 6
      IF(JCODE.LE.2) JVP=JPAR
      IF(JCODE.EQ.3) JVP=KPAR
  301 CONTINUE
  205 NAUX=ND(JVP)
      XT=X(NAUX)
      DO 100 JK=1,30
      CALL CONTROL(2,2,2,4)
      K=0
      DO 200 I=1,N
      NAUX=KT(I)
      W(I)=-AF(NAUX)
      IF(ABS(AF(NAUX)).GE.TOLCON)K=1
 200  CONTINUE
      IF(K.EQ.0) GO TO 300
      DO 201 I=1,N
      NAUX=KT(I)
      DO 201 J=1,N
      NAUXT=NS(J)
      AMA(I,J)=ADFDX(NAUX,NAUXT)
 201  CONTINUE
      IF(N.EQ.1) V(1)=W(1)/ADFDX(NAUX,NAUXT)
      IF(N.EQ.1) GO TO 10
      NDESIZ=N
      CALL GAUSS
   10 CONTINUE
      DO 300 I=1,N
      NAUX=NS(I)
      X(NAUX)=X(NAUX)+V(I)
      IF(IFREE(NAUX).GT.0.AND.X(NAUX).LT.0.0) X(NAUX)=0.0
 300  CONTINUE
 100  CONTINUE
      KP=KP+1
      IF(KP.GT.10) GO TO 102
      ICODE=-[illegible]
      IF(JCODE.EQ.3) GO TO 102
      DO 206 I=1,N
      NAUX=NS(I)
  206 X(NAUX)=XI(NAUX)
      NAUX=ND(JVP)
      X(NAUX)=XT+(XI(NAUX)-XT)/2.0
      GO TO 301
  102 WRITE(6,109)
 109  FORMAT([illegible])
      [illegible]
      CALL [illegible]
  [illegible] IF(IPRINT.GT.0) GO TO 6
C     WRITE(6,8)
    8 FORMAT([illegible]
     1[illegible])
      WRITE(6,101) [illegible]
  101 FORMAT([illegible])
C     RETURN
      END
```

CODE LISTING OF GAUSS

```
      SUBROUTINE GAUSS
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTR,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NCI(30),
     1ND(30),NS(30),NT(30),NL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      DIMENSION TEMP(30)
      IX=1
      DO 1 I=1,NDESIZ
      TEMP(I)=0.
      V(I)=0.0
    1 CONTINUE
      NN=NDESIZ-1
      DO 30 K=1,NN
   19 AKK=AMA(K,K)
      KK=K+1
      DELTA=ABS(AKK)
      IF(DELTA.GT.TOLJAC) GO TO 18
      IMAX=NDESIZ-K
      DO 20 L=1,IMAX
      KPL=K+L
      DELTA=ABS(AMA(KPL,K))
      IF(DELTA.LE.TOLJAC) GO TO 20
      AUX=W(K)
      W(K)=W(KPL)
      W(KPL)=AUX
      DO 21  J=1,NDESIZ
      TEMP(J)=AMA(K,J)
      AMA(K,J)=AMA(KPL,J)
      AMA(KPL,J)=TEMP(J)
   21 CONTINUE
      GO TO 17
   20 CONTINUE
      GO TO 40
   17 IX=IX*(-1)
   18 DO 30 I=KK,NDESIZ
      BB=W(K)
      AKK=AMA(K,K)
      AIK=AMA(I,K)
      IF(ABS(AKK).LE.TOLJAC) AKK=0.0
      IF(ABS(AKK).LE.TOLJAC) GO TO 120
      AA=AIK/AKK
      W(I)=W(I)-AA*BB
  120 CONTINUE
      DO 30 M=K,NDESIZ
      AMA(I,M)=AMA(I,M)-AA*AMA(K,M)
   30 CONTINUE
   40 CONTINUE
      DO 50   ICOMP=1,NDESIZ
      I=NDESIZ+1-ICOMP
      SUM=0.0
      DO 60 L=1,NDESIZ
      SUM=SUM+AMA(I,L)*V(L)
   60 CONTINUE

      IF(ABS(AMA(I,I)).LE.TOLJAC) V(I)=0.0
      IF(ABS(AMA(I,I)).LE.TOLJAC) GO TO 50
      V(I)=(W(I)-SUM)/AMA(I,I)
   50 CONTINUE
      DETERM=1.0
      DO 80 I=1,NDESIZ
      DETERM=DETERM*AMA(I,I)
   80 CONTINUE
      DETERM=DETERM*FLOAT(IX)
   70 RETURN
      END
```

CODE LISTING OF DATAOUT

```
      SUBROUTINE DATAOUT(ITITLE,ISUBTIT,NAME,IYOFX,ICTYPE,     NFREEVA)
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTK,NKEQ,NLK,IREQ,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /T0L/ T0LCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),KT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
C
      DIMENSION ITITLE(8),ISUBTIT(8),NAME(8)
C
      WRITE(6,80)(ITITLE(I),I=1,8)
 80   FORMAT(1H1,10X,8A10/)
      WRITE(6,81)(ISUBTIT(I),I=1,8)
 81   FORMAT(1H0,10X,8A10/)
      WRITE(6,82)(NAME(I),I=1,8)
 82   FORMAT(1X,10X,8A10//)
      WRITE(6,83) NORIVA
 83   FORMAT(1X,10X,*NUMBER OF ORIGINAL VARIABLES IS*,4X,I3)
      WRITE(6,84) NFREEVA
 84   FORMAT(1X,10X,*NUMBER OF FREE VARIABLES IS*,8X,I3)
      WRITE(6,85) NKEQ
 85   FORMAT(1X,10X,*NUMBER OF EQUALITY CONSTRAINTS IS*,2X,I3)
      WRITE(6,86) NKINEQ
 86   FORMAT(1X,10X,*NUMBER OF INEQUALITY CONSTRAINTS IS*I3//)
C
      IF(IYOFX-2)87,88,89
 87   WRITE(6,20)
 20   FORMAT(1X,20X,*THE OBJECTIVE FUNCTION IS LINEAR*)
      GO TO 47
 88   WRITE(6,21)
 21   FORMAT(1X,20X,*THE OBJECTIVE FUNCTION IS QUADRATIC*)
      GO TO 47
 89   WRITE(6,22)
 22   FORMAT(1X,10X,*THE OBJECTIVE FUNCTION IS NONLINEAR*)
 47   IF(ICTYPE-1)23,24,23
 23   WRITE(6,25)
 25   FORMAT(1H0,10X,*SOME CONSTRAINTS ARE NONLINEAR*)
      GO TO 48
 24   WRITE(6,26)
 26   FORMAT(1H0,10X,*ALL CONSTRAINTS ARE LINEAR*)
 48   J=1
      DO 30 I=1,NORIVA
      IF(IFREE(I)-1)27,30,30
 27   ND(J)=I
      J=J+1
 30   CONTINUE
      J1=J-1
      IF(J1.EQ.0) GO TO 100
      WRITE(6,31)(ND(I),I=1,J1)
 31   FORMAT(1H0,10X,*THE FREE VARIABLES OF THE PROBLEM ARE THE ORIGINAL
     1VARIABLES WITH INDICES=*/30(I2,*,*))
      GO TO 102
 100  WRITE(6,101)
 101  FORMAT(1H0,10X,*THERE ARE NO FREE VARIABLES*)
  102 CONTINUE
```

```
      WRITE(6,46) (X(N),N=1,NORIVA)
   46 FORMAT(1H0,10X,*INITIAL FEASIBLE SET OF VARIABLES*//5X,1H(,20(F5.1
     1,1X),1H))
   49 WRITE(6,50)TOLGON ,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLY
   50 FORMAT(1H0,10X,*TOLERANCES*//,11X,*TOLERANCE ON THE PROBLEM CONSTR
     1AINTS*,4X,F10.6/11X,*TOLERANCE ON STATE VARIABLES*11X,F10.6/11X,*T
     2OLERANCE ON THE JACOBIAN MATRIX*,8X,F10.6/11X,*TOLERANCE ON CONSTR
     3AINED DERIVATIVES*3X,F10.6/11X,*TOLERANCE ON DECISION VARIABLES*,9
     4X,F10.6/11X,*TOLERANCE ON THE OBJECTIVE FUNCTION*,5X,F10.6/)
      WRITE(6,60) MAXLEV,MAXITER
   60 FORMAT(1H0,9X,*FREQUENCY OF COMPUTATIONAL OUTPUTS                *
     1,17X,I5/10X,*MAX NUMBER OF ITERATIONS IN THE OPTIMIZATION PROCESS*
     2,15X,I5/)
      WRITE(6,9999)
 9999 FORMAT(1H1)
      RETURN
      END
```

CODE LISTING OF ANSOUT

```
      SUBROUTINE ANSOUT
      COMMON /CONT/ NORIVA,NKINEQ,NKTOT,MAXITER,IPRINT,MAXLEV,NDESIZ,
     1NSV,NDV,NTR,NKEQ,NLK,IREG,IMIN,JPAR,KPAR,ICODE,IPARP,KPARP
      COMMON /TOL/ TOLCON,TOLONS,TOLJAC,TOLVJ,TOLDJ,TOLDIP,TOLY,TOLKUN
      COMMON /STATE/ Y,AJACOBI,DETERM,AF(30),X(30),  UT(30,30),NC1(30),
     1ND(30),NS(30),NT(30),KL(30),IFREE(30)
      COMMON /DERIV/ ADYDX(30),ADFDX(30,30),ADYDS(30),ADYDD(30),ADFTDS(3
     10,30),ADFTDD(30,30),ADFLDS(30,30),ADFLDD(30,30)
      COMMON /CODRIV/ CODYDD(30),CODYDF(30),CODSDD(30,30),CODSDF(30,30),
     1CODFLDD(30,30),CODFLDF(30,30)
      COMMON /MISC/ W(30),V(30),AMA(30,30)
      COMMON JOBCODE
      CALL CONTROL(1,2,1,1)
      IF(JOBCODE.EQ. 1) GO TO 40
      IF(JOBCODE.EQ. 3) GO TO 40
      IF(JOBCODE.EQ. 5) GO TO 40
      IF(JOBCODE.EQ. 7) GO TO 40
      IF(JOBCODE.EQ. 9) GO TO 40
      IF(JOBCODE.EQ.11) GO TO 40
      IF(JOBCODE.EQ.13) GO TO 40
      CALL KODHIV(1,2,2,2,2,2)
      WRITE(6,1)
    1 FORMAT(1H1,25X,*RESULTS OF THE OPTIMIZATION WITH THE GEN. DIF. ALG
     1ORITHM*)
      WRITE(6,2)
    2 FORMAT(///,37X,*BY  -   W.H.WALKER*)
      M=NORIVA-NSV
      WRITE(6,4)Y
    4 FORMAT(////,25X,*VALUE OF OBJECTIVE FUNCTION AT MINIMUM POINT.  Y
     .=*,F20.4)
      WRITE(6,5)
    5 FORMAT(//,25X,*VALUES OF VARIABLES, X, AT OPTIMAL POINT.*,//,30X,*
     .STATE VARIABLES -*)
      IF(NSV.EQ.0) WRITE(6,100)
  100 FORMAT(1H0,25X,*PROBLEM IS UNCONSTRAINED*)
      IF(NSV.EQ.0) GO TO 101
      DO 50 J=1,NSV
      NAUX=NS(J)
      AUX=X(NAUX)
      WRITE(6,6)J,NAUX,AUX
    6 FORMAT(30X,*S(*,I2,*) = X(*,I2,*) =*,F20.4)
      IF(IFREE(NAUX).GT.0) GO TO 50
      WRITE(6,7)
    7 FORMAT(*+*,70X,*(FREE)*)
   50 CONTINUE
  101 CONTINUE
      WRITE(6,8)
    8 FORMAT(/,30X,*DECISION VARIABLES -*)
      DO 55 J=1,NDV
      IF(J.GT.M) GO TO 55
      NAUX=ND(J)
      AUX=X(NAUX)
      WRITE(6,9)J,NAUX,AUX
    9 FORMAT(30X,*D(*,I2,*) = X(*,I2,*) =*,F20.4)
      IF(IFREE(NAUX).GT.0) GO TO 55
      WRITE(6,7)
   55 CONTINUE
      WRITE(6,10)
   10 FORMAT(//,25X,*VALUES OF CONSTRAINED DERIVATIVES.*,//,30X,*DY/DD =
     .*)
      IF(M.LE.0) GO TO 41
      DO 60 J=1,M
      WRITE(6,11) CODYDD(J)
   11 FORMAT(35X,F20.4)
   60 CONTINUE
      WRITE(6,12)
   12 FORMAT(/,30X,*DY/DF =*)
   41 CONTINUE
      IF(NSV.EQ.0) GO TO 102
      DO 65 J=1,NSV
      WRITE(6,11) CODYDF(J)
   65 CONTINUE
      IF(NKEQ.EQ.0) GO TO 102
      WRITE(6,13)
   13 FORMAT(//,25X,*VALUES OF CONSTRAINTS.*,//,30X,*EQUALITIES -*)
      DO 70 J=1,NKEQ
      NAUX=KT(J)
      AUX=AF(NAUX)
      WRITE(6,14)J,NAUX,AUX
   14 FORMAT(30X,*F(*,I2,*) = AF(*,I2,*) =*,F20.4)
   70 CONTINUE
  102 CONTINUE
      IF(NKEQ.EQ.NSV) WRITE(6,92)
      IF(NKEQ.EQ.NSV) GO TO 91
   92 FORMAT(1H0,30X,*NONE OF THE INEQUALITIES ARE TIGHT*)
      WRITE(6,15)
   15 FORMAT(//,30X,*VALUES OF TIGHT CONSTRAINTS -*)
      J1=NKEQ+1
      DO 75 J=J1,NSV
      NAUX=KT(J)
      AUX=AF(NAUX)
      WRITE(6,14)J,NAUX,AUX
   75 CONTINUE
   91 MP=NKINEQ+NKEQ
      IF(NSV.GE.MP) WRITE(6,93)
      IF(NSV.GE.MP) GO TO 94
   93 FORMAT(1H0,30X,*NONE OF THE INEQUALITIES ARE LOOSE*)
      WRITE(6,16)
   16 FORMAT(//,30X,*VALUES OF LOOSE CONSTRAINTS -*)
      DO 80 J=1,NLK
      NAUX=KL(J)
      AUX=AF(NAUX)
      M=J+NKEQ+NTK
      WRITE(6,14)M,NAUX,AUX
   80 CONTINUE
   94 CONTINUE
      WRITE(6,17)
   17 FORMAT(////,25X,*THE ABOVE VALUES ARE PRINTED OUT BECAUSE -*)
      IF(IMIN.NE.1) GO TO 30
      WRITE(6,18)
   18 FORMAT(/,25X,*KUHN-TUCKER CONDITIONS ARE SATISFIED*)
      GO TO 40
   30 IF(IMIN.NE.2) GO TO 32
      WRITE(6,19)
   19 FORMAT(/,25X,*THE NUMBER OF PRESCRIBED ITERATION STEPS ARE EXCEEDE
     .D.*)
      GO TO 40
   32 IF(IMIN.NE.3) GO TO 34

      WRITE(6,20)
   20 FORMAT(/,25X,*THE REDUCTION IN OBJECTIVE FUNCTION IS TOO SMALL.*)
      GO TO 40
   34 IF(IMIN.NE.4) GO TO 40
      WRITE(6,21)
   21 FORMAT(/,25X,*FUNCTIONAL PROBLEM IN PROGRAM, SEE POINT OF RELEASE*
     1)
   40 CONTINUE
      IMIN=10
      RETURN
      END
```